Bottom friction estimation in the context of regional barotropic tide modelling

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Introduction

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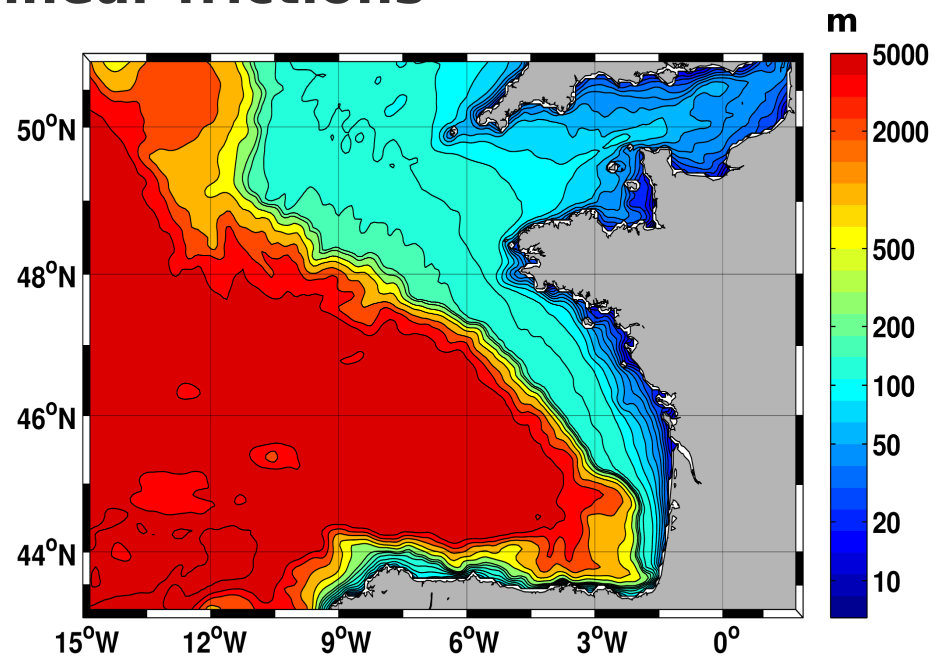
Study area



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Quadratic friction coefficient computation based on vertical integration of the turbulent velocity profile:

$$C_B = \left(\frac{\kappa}{\ln \frac{H}{z_0} - 1} \right)^2$$

κ : Von Kármán constant
z_0: bottom roughness
H : total water height

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Method performances evaluated with twin experiments

Optimization procedure

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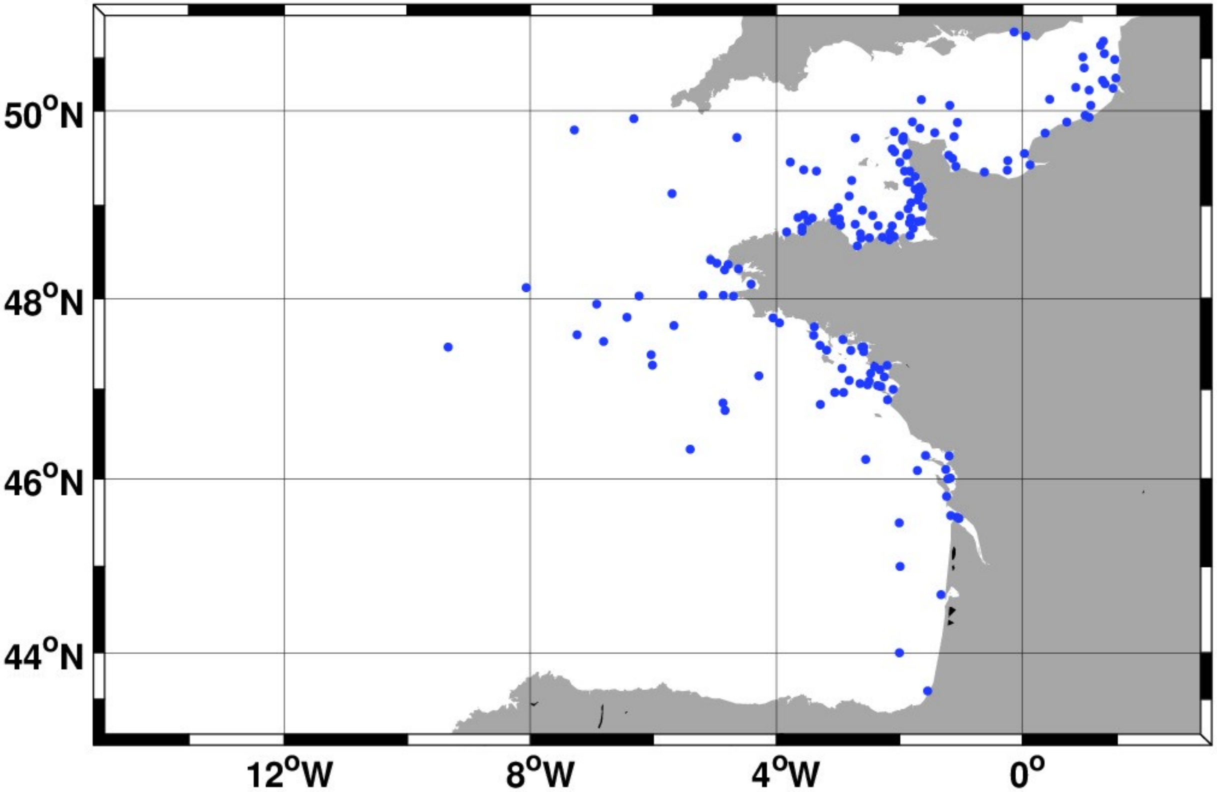

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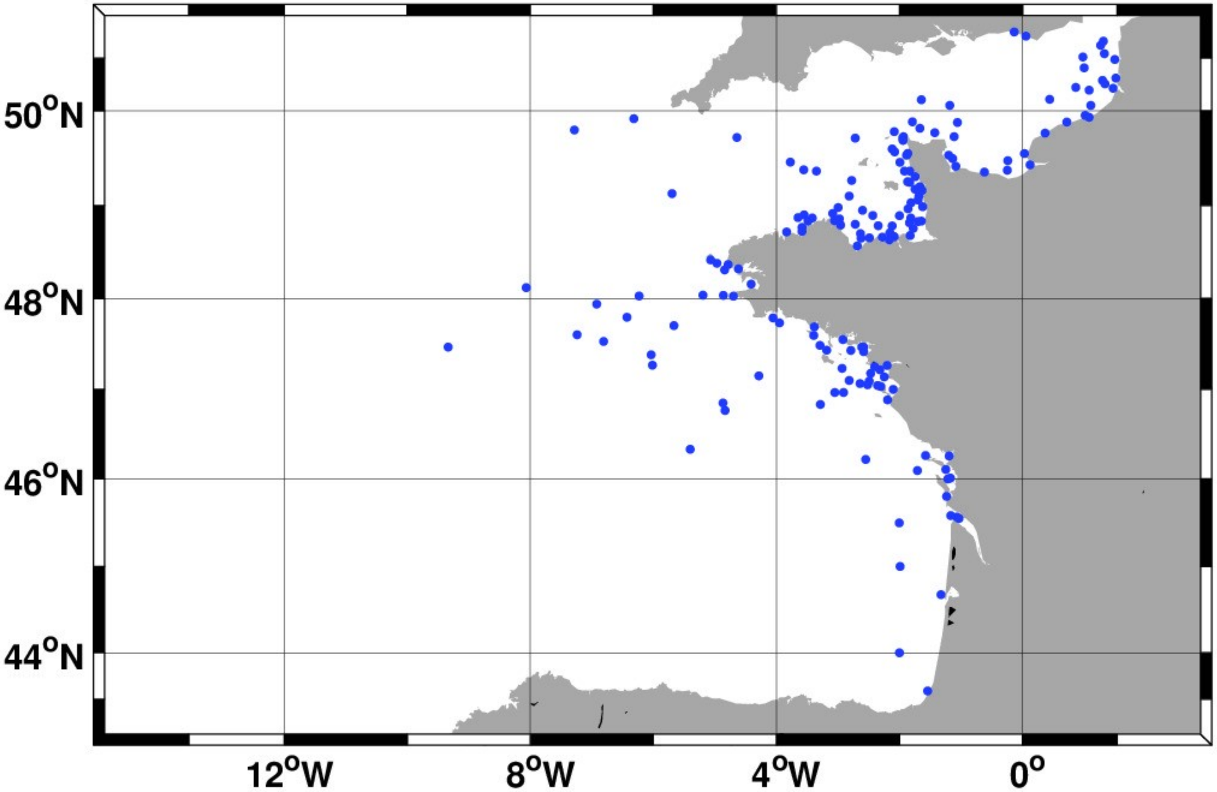

- **Tidal spectrum:**
  - **- Astronomic components: M2 and S2**
  - **- Non-linear components of M2 and S2: semidiurnal, quarter-diurnal, ..., eighth-diurnal species**

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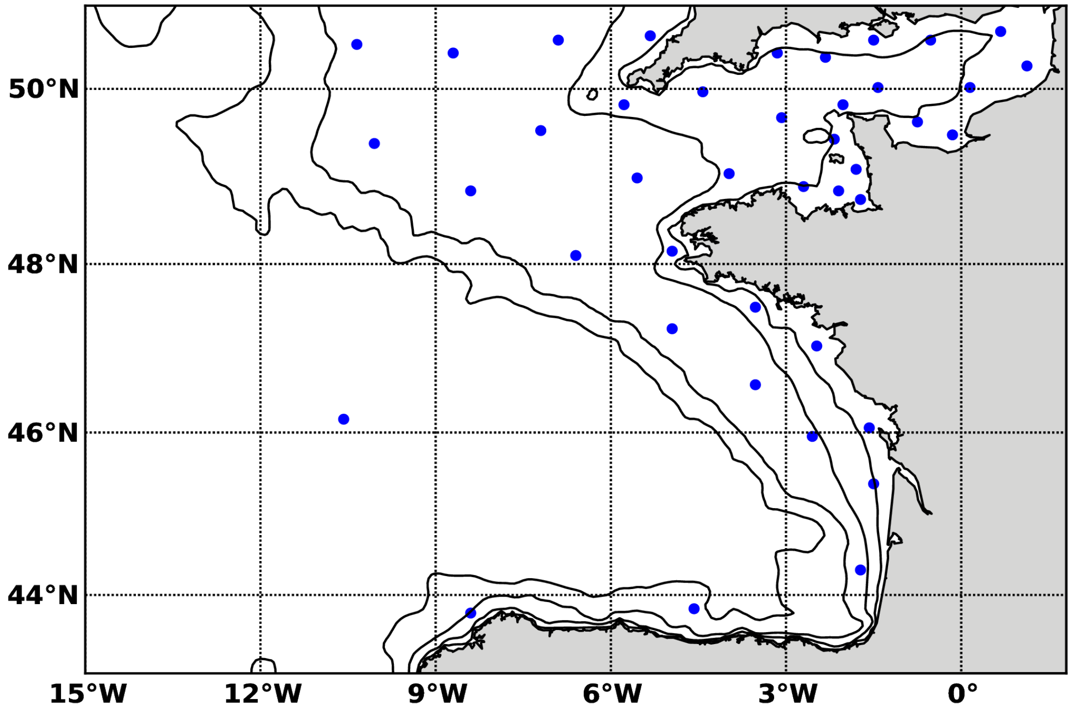

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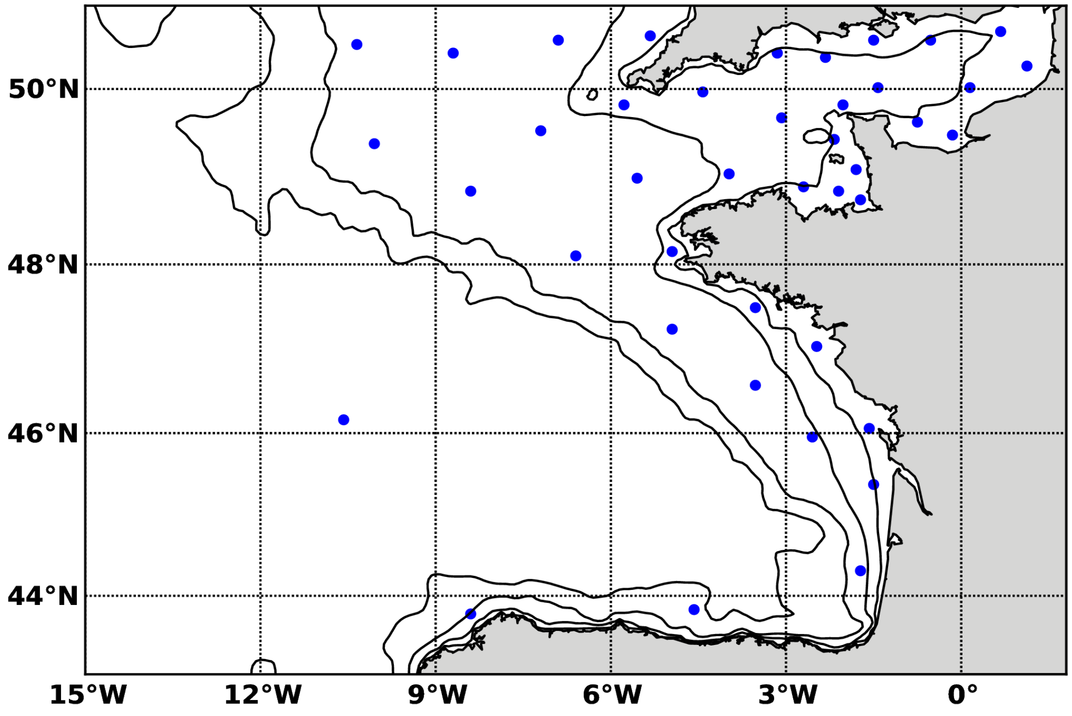

2D reconstruction using interpolation

- **Cost function computed over a spring/neap cycle**

$$J = \frac{1}{2} \sum_{t=1}^{T} \sum_{n=1}^{N} (SSH_{n,t}^{mod} - SSH_{n,t}^{obs})^2$$

Purely quadratic friction: roughness estimation

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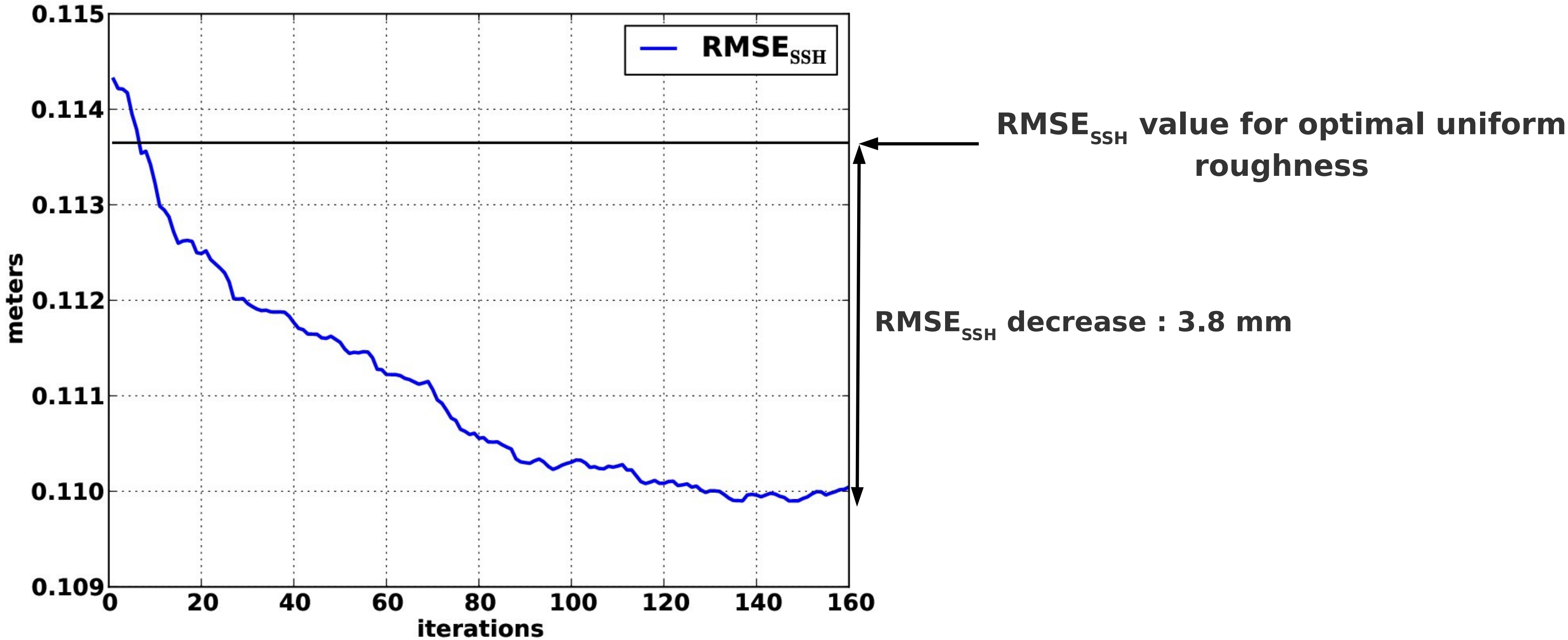

Root-mean square error against tide gauge observations

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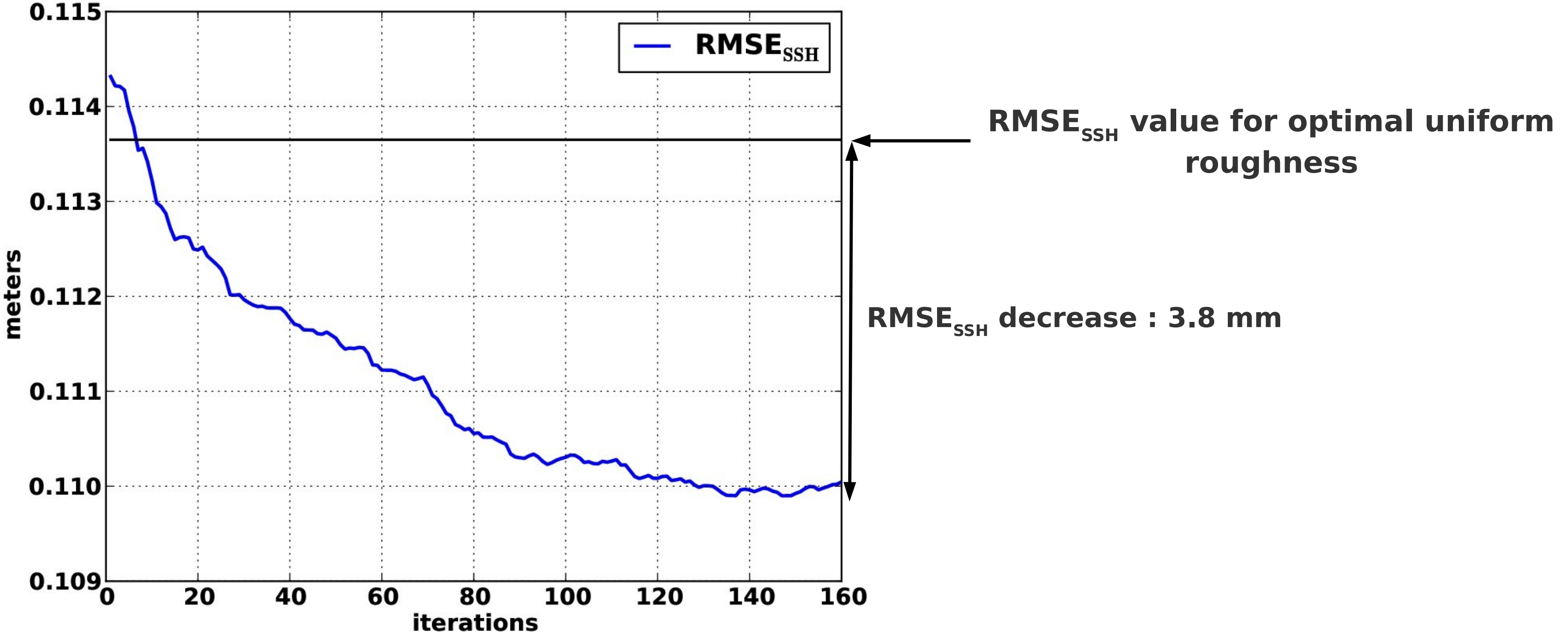

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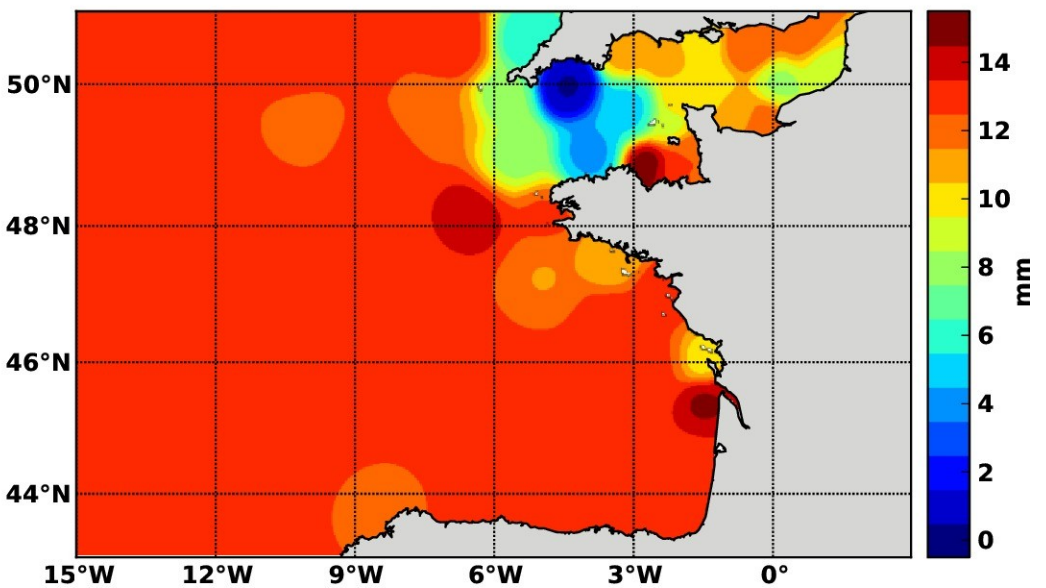
Roughness distribution



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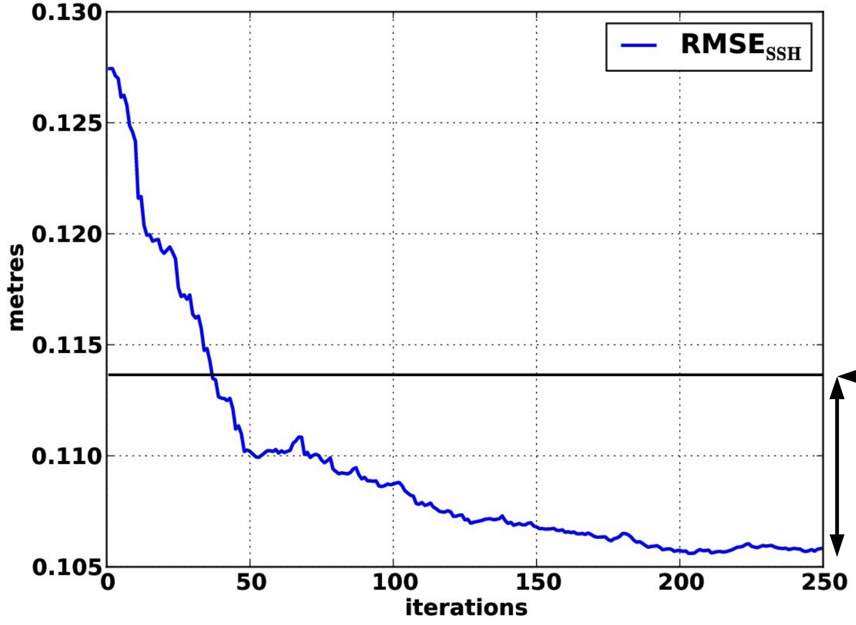

RMSE$_{SSH}$ value for optimal uniform roughness and linear friction coefficient

RMSE$_{SSH}$ decrease: 8 mm

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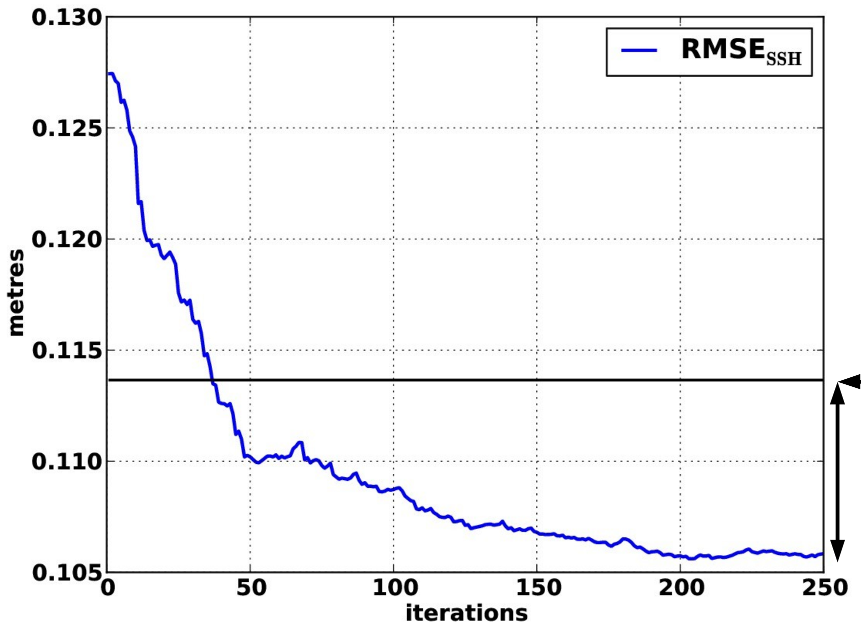

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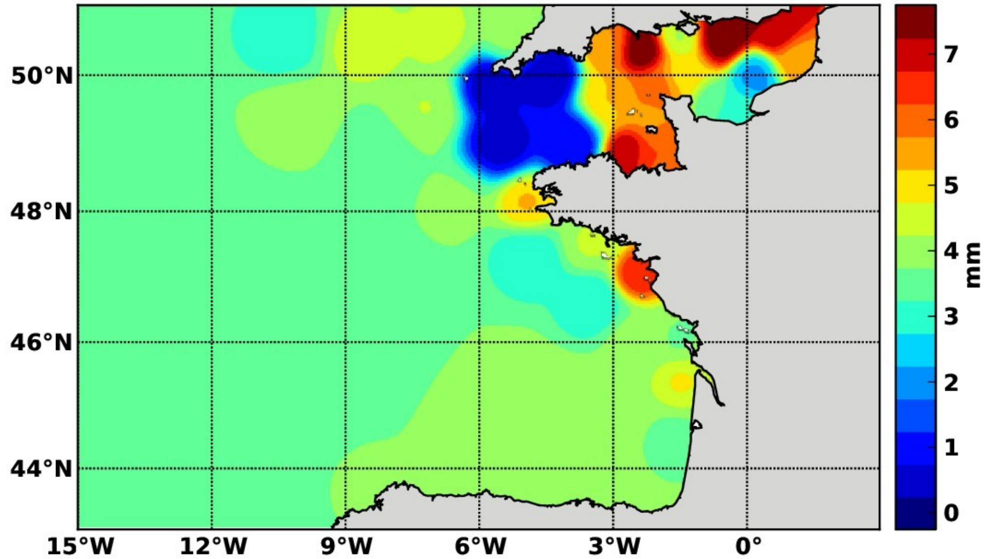

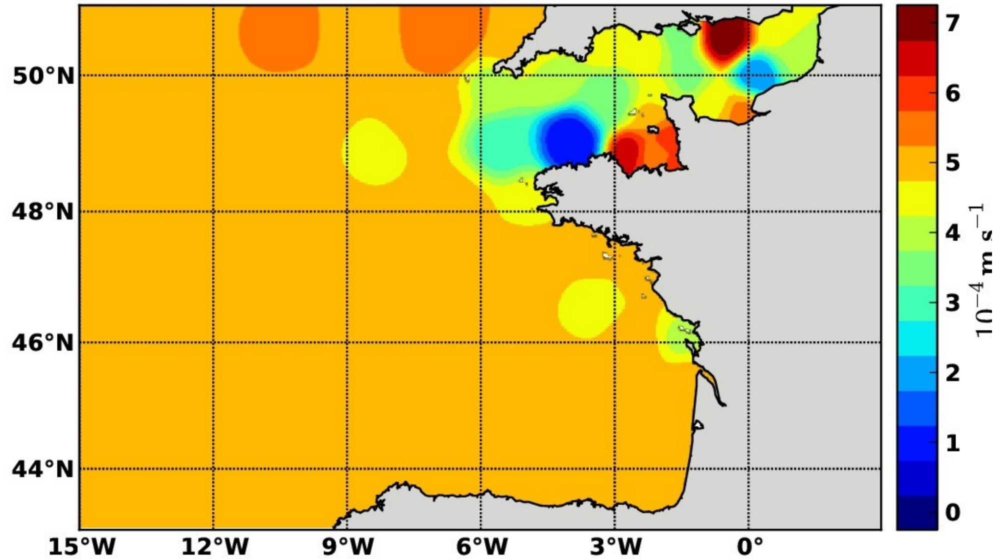

Error variations before and after optimization

Error variations over a spring/neap cycle: six tidal ranges

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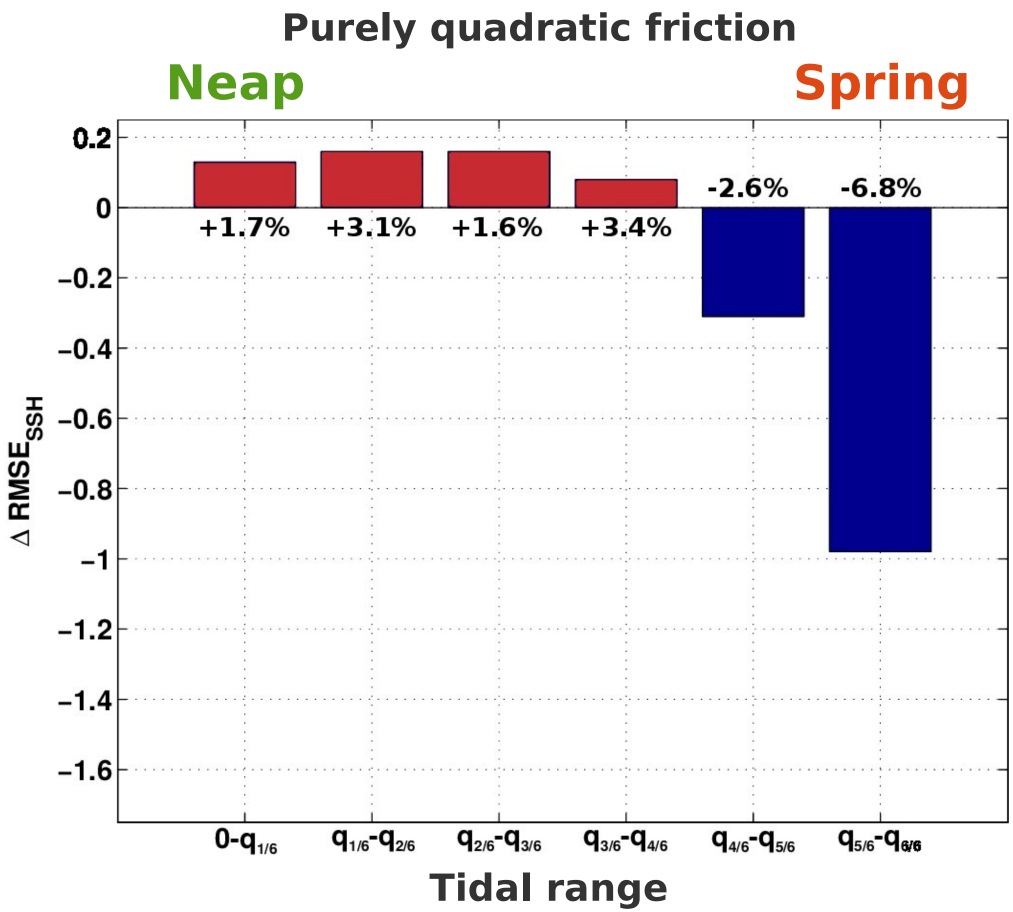

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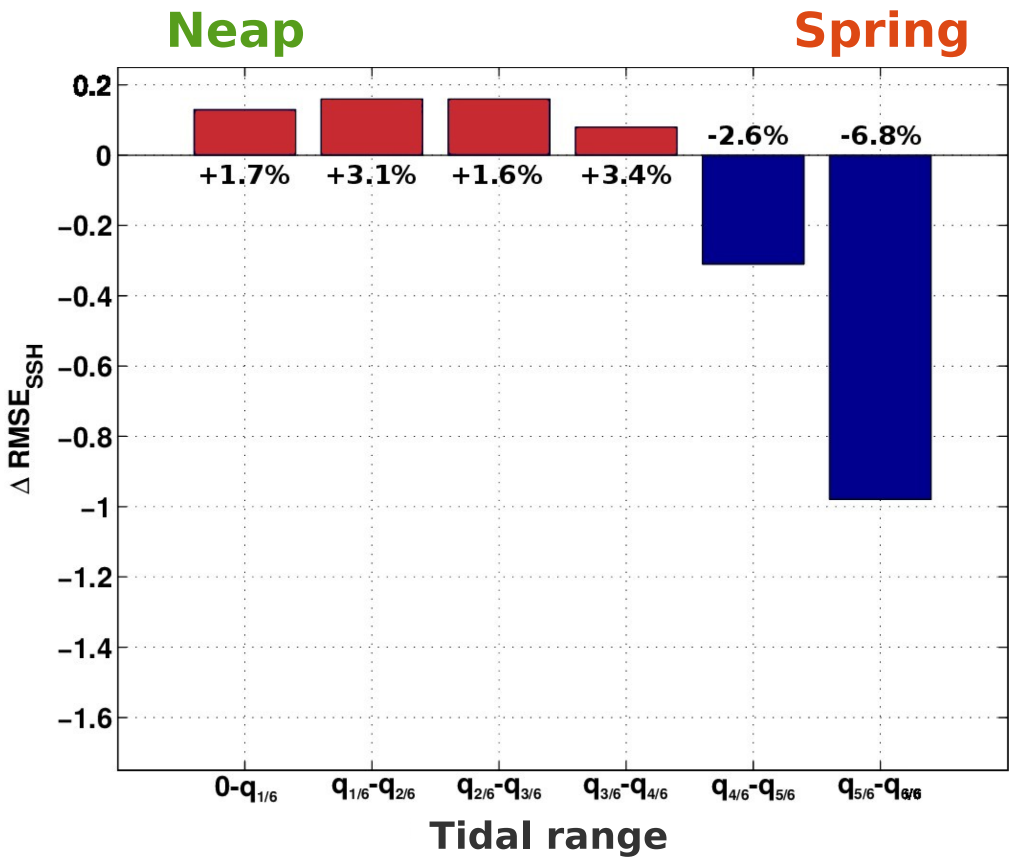
Purely quadratic friction



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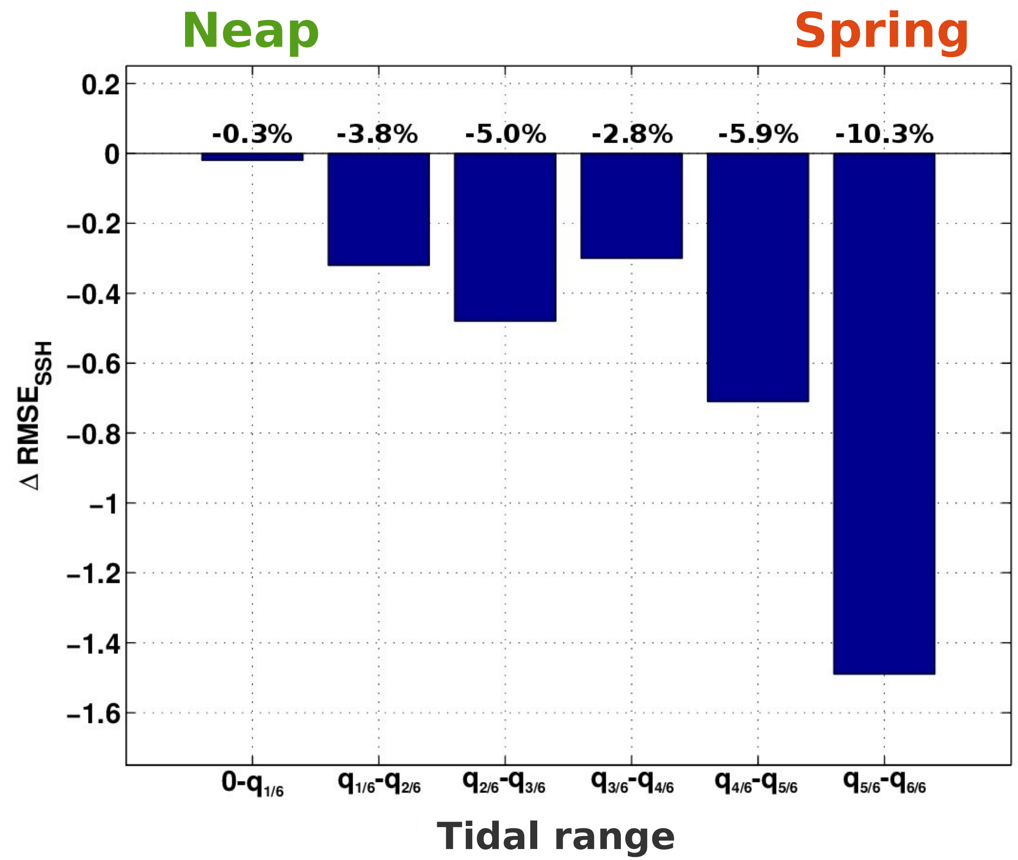

Conclusions and perspectives

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- **Residual errors**
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PERSPECTIVES

- Assimilation of current data: ADCP profiler, HF radars
- Open boundary condition optimization
- Increase the number of tidal components, especially the diurnal species
- Extend this work to a baroclinic model