An Integrated Energetics Approach to Determining Planetary Boundary Layer Mixing

Robert Hallberg

with contributions from Alistair Adcroft and Steve Griffies

NOAA / GFDL and Princeton University

Approaches to Planetary Boundary Layer Mixing in Global Ocean Models

1. Bulk mixed layer (Niiler and Kraus, 1977; Bleck et al., 1989; Oberhuber, 1993; Hallberg, 2003; etc.):

   Mix infinitely quickly over a near-surface region whose extent is determined by an implicit integrated Turbulent Kinetic Energy (TKE) budget.

2. 2-equation closures (Mellor & Yamada, 1974; Umlauf and Buchard, 2005):

   Solve explicit equations for the evolution of TKE & dissipation or mixing length.

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial z}\left(\frac{\nu}{\sigma_k}\frac{\partial k}{\partial z}\right) + \nu S^2 - \kappa N^2 - \varepsilon \qquad \frac{\partial \varepsilon}{\partial t} = \frac{\partial}{\partial z}\left(\frac{\nu}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial z}\right) + \frac{\varepsilon}{k}\left(c_1 \nu S^2 - c_3 \kappa N^2 - c_2 \varepsilon\right) \qquad (\nu, \kappa) = (c_\mu, c'_\mu)(c_\mu^0)^3 \frac{k^2}{\varepsilon}$$

3. Explicit TKE equation with prescribed mixing length (Gaspar et al., 1990): Solve explicit equations for the evolution of TKE & use a parametric mixing length-scale.

4. K-Profile Parameterization (KPP) (Large et al., 1995):

   Specify a diffusivity profile based on a diagnosed boundary layer depth and a turbulent velocity.

$$Ri_{Bulk}(H_{BL}) = Ri_{Crit} \qquad \kappa(z) = 0.4\sqrt{u^{*2} + w^{*2}}\, z\left(1 + (z/H_{BL})^2\right)$$

5. Energetics Planetary Boundary Layer (ePBL; NEW SCHEME):

   Solve an implicit integrated TKE budget in a single downward pass (like a bulk mixed layer) using a diffusivity profile based on a turbulent velocity and parametric mixing length-scale (like KPP or Gaspar)

K-Profile Parameterization (Large et al., Rev. Geophys., 1994.)

Specification of boundary layer diffusivities based on empirical principles for the boundary layer depth, dimensional analysis, and assumed structures.

- Bulk Richardson number determines the boundary layer depth.
- Non-local (non-diffusive) buoyancy fluxes are specified within the mixed layer arising from penetrating convection.

$$Ri_{Bulk}(z) = \frac{g|z|\left(\rho(z) - \bar{\rho}^{0,z}\right)}{\left\|\vec{u}(z) - \bar{\vec{u}}^{0,z}\right\|^2}$$

$$Ri_{Bulk}(z) > Ri_{Bulk,Crit} \forall |z| < |H_{BL}|$$

- Monin-Obukhov similarity theory and Law of the wall are used in the KPP diffusivity specification:

$$\kappa(z) = k_{Karman}\sqrt{u^{*2} + w^{*2}}\, H_{BL} F\left(\frac{z}{H_{BL}}\right) StabilityFns(\ldots)$$

$$u^{*2} = \frac{\|\vec{\tau}\|}{\rho} \qquad w^{*3} = BH_{BL}$$

$$F(x) = x(1-x)^2$$

Issues with KPP:

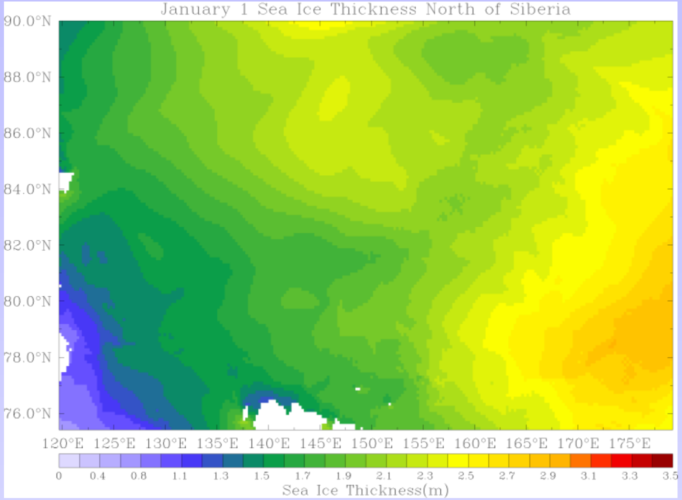
- Solution is very sensitive to choices about interpolation and matching.
- Explicit boundary-layer depth determination leads to instabilities.
- Nonlocal heat and salinity fluxes can violate the second law of thermodynamics.
- No constraints on the energy consumed in mixing.
- Expressions also include additional empirical “structure functions” (not shown above).
- Mechanically forced turbulence does not require a small resolved Richardson number!
- Appropriate for internally generated boundary layers (like the atmosphere), but less so for an externally forced boundary layer turbulence (like the ocean)!

noaaresearch

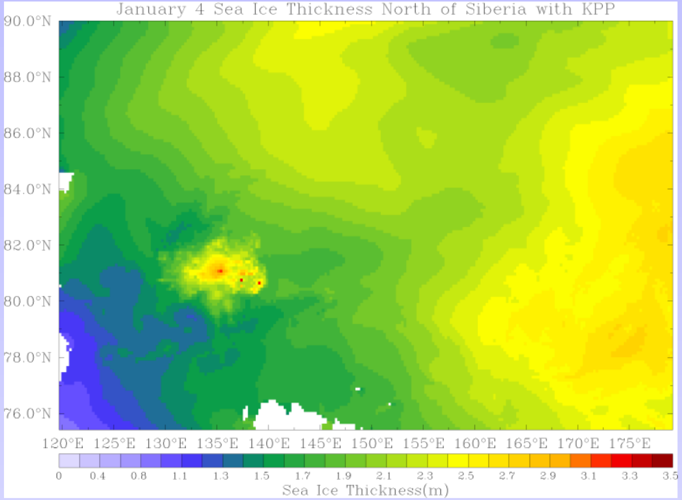
GFDL

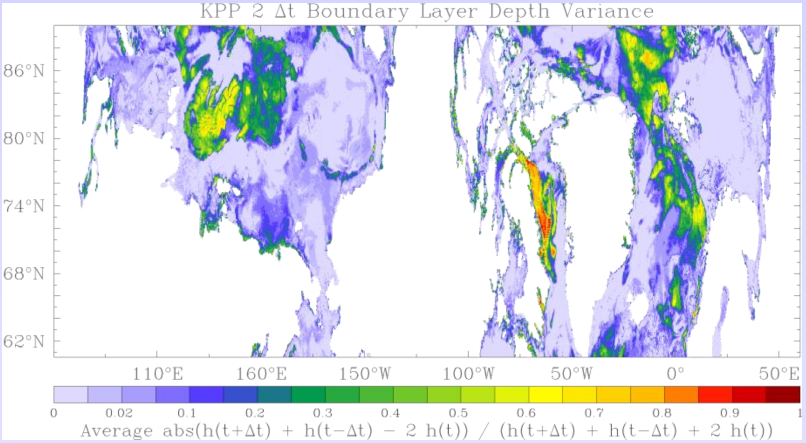
Explosive Sea-Ice Growth as a Manifestation of Instability in KPP

Jan. 1 Sea Ice Thickness of Siberia

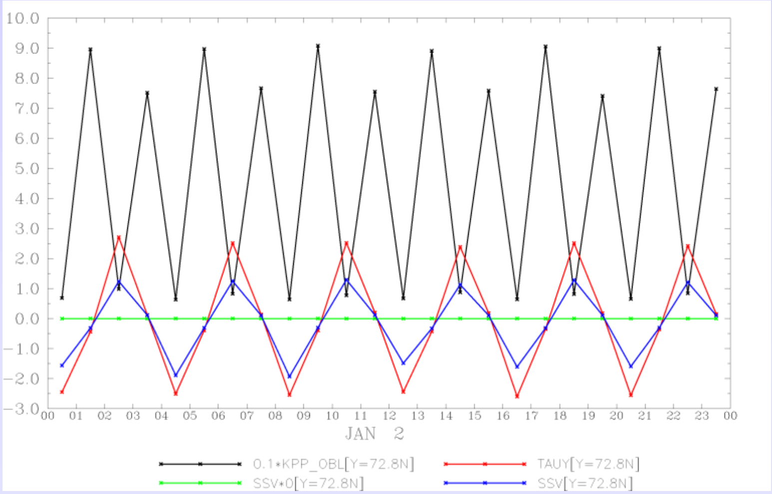


Jan. 4 Sea Ice Thickness of Siberia

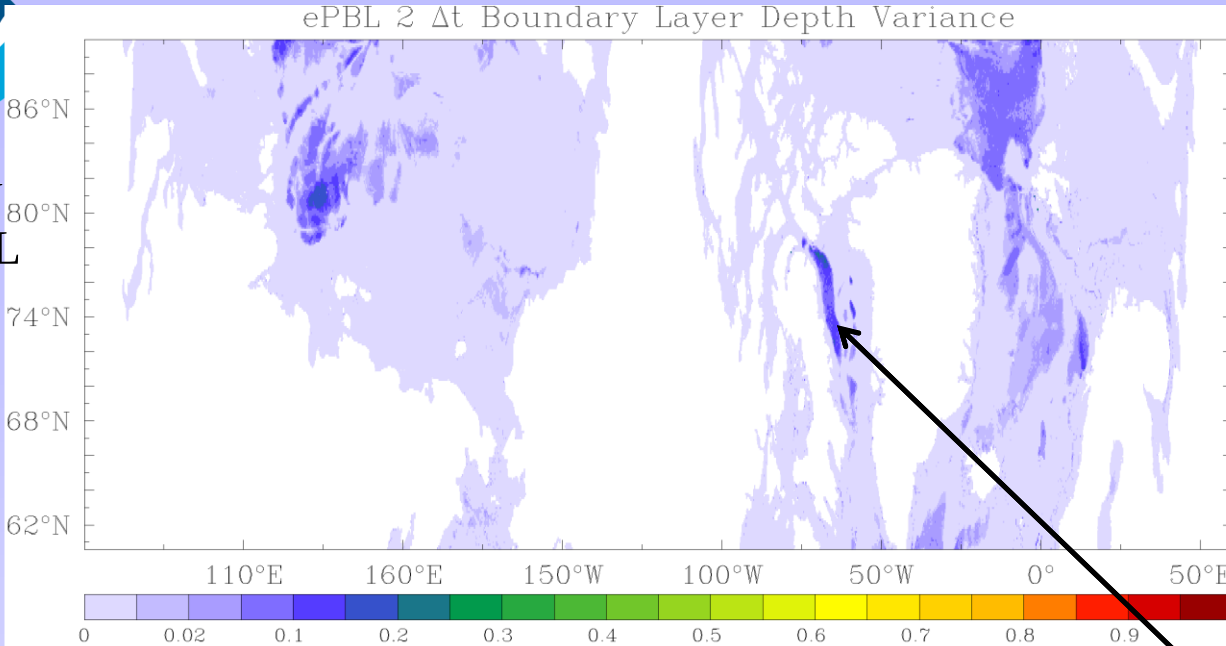




$$\overline{\left|2h_t - h_{t-\Delta t} - h_{t+\Delta t}\right| / \left(2h_t + h_{t-\Delta t} + h_{t+\Delta t}\right)}$$

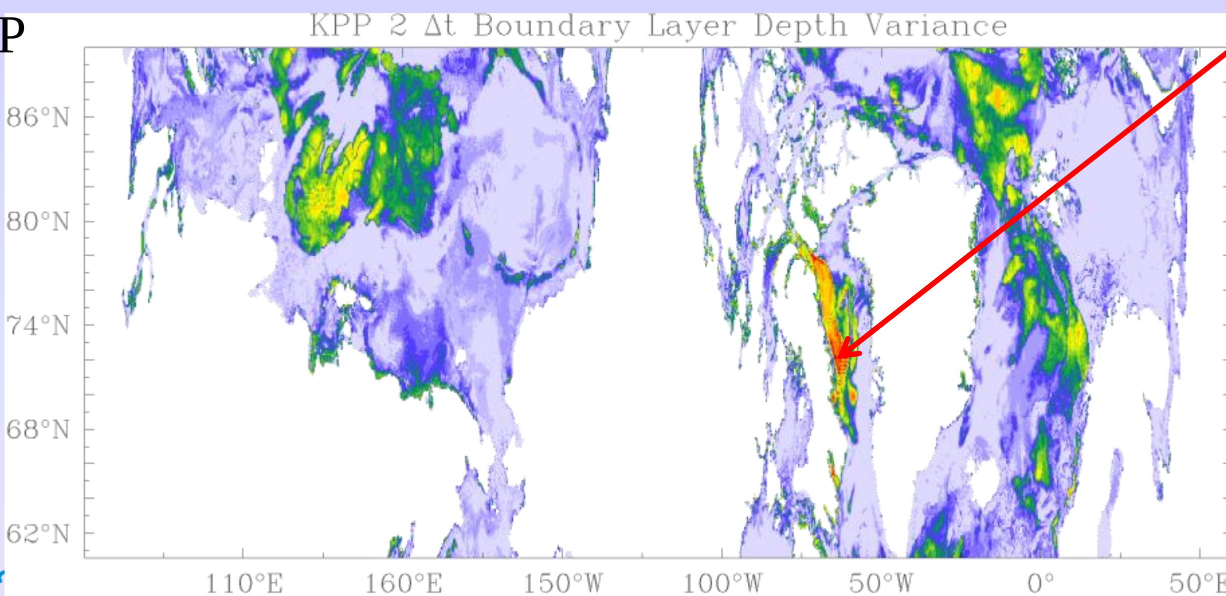


New ePBL

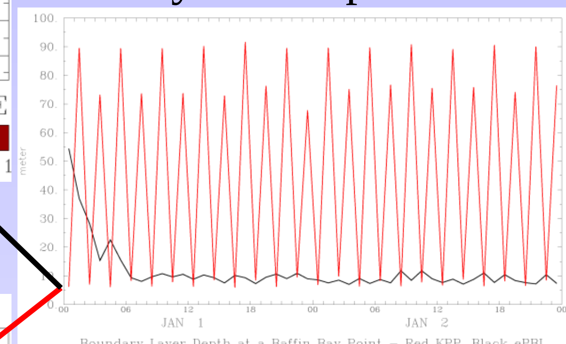


$$\overline{\left|2h_t - h_{t-\Delta t} - h_{t+\Delta t}\right| / \left(2h_t + h_{t-\Delta t} + h_{t+\Delta t}\right)}$$

KPP



Boundary Layer Depth at a Baffin Bay Point, Every timestep shown

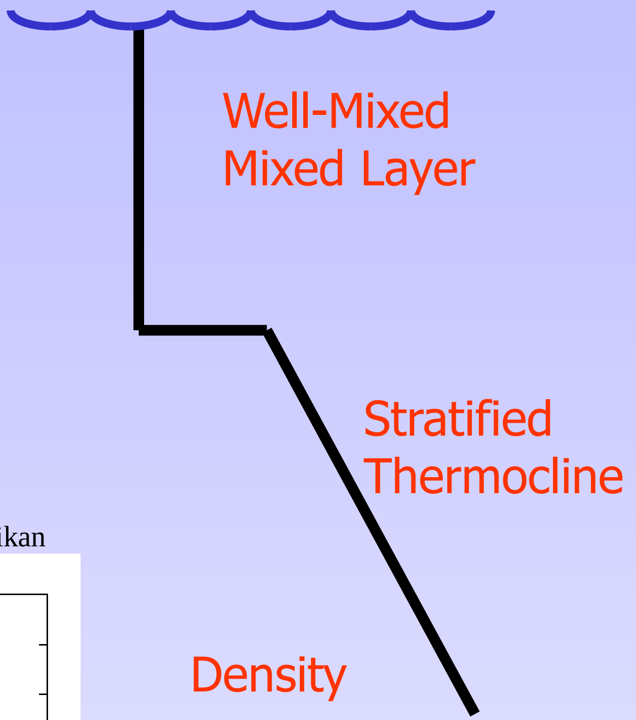


A nice day in the Tasman Sea: the ocean being mixed!

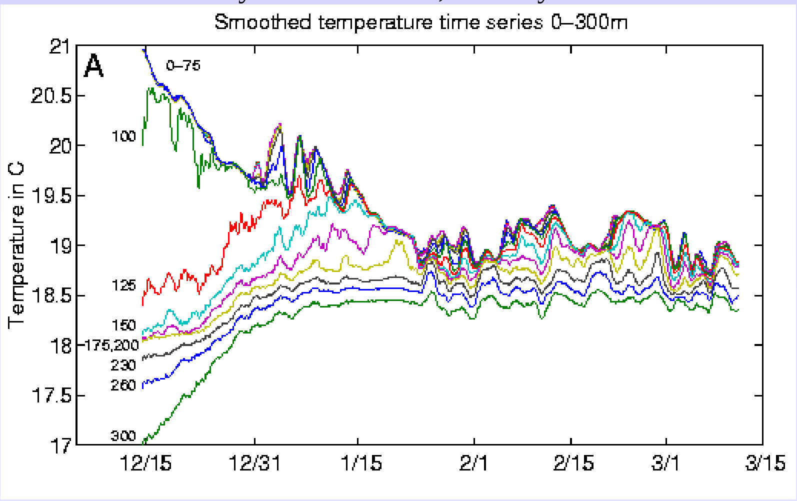
Movie taken from R.V. Revelle by R. Hallberg.

Assumptions underpinning the Kraus-Turner bulk mixed Layer Model

1. Temperature, salinity and horizontal velocity are perfectly well mixed in the surface boundary layer.
2. A quasi-discontinuous profile can be used across the base of the mixed layer.
3. Turbulence is in instantaneous balance (sources and sinks dominate storage) in a vertically integrated TKE budget, and stops abruptly at the mixed layer base.



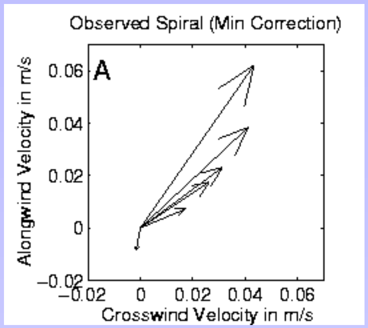
ASREX Mixed Layer Observations, courtesy A. Gnanadesikan

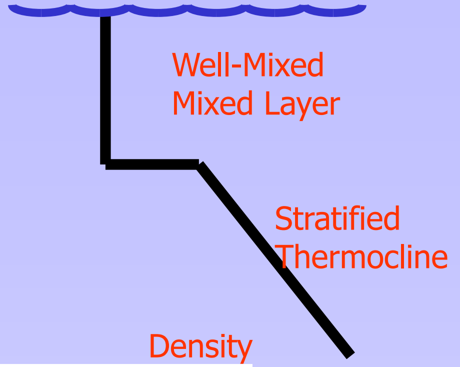


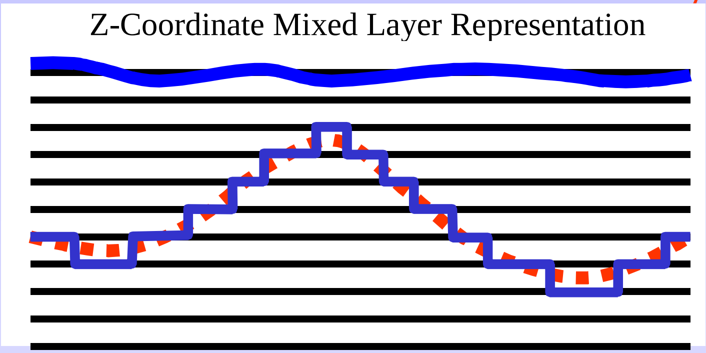
Kraus & Turner, Tellus 1967; Niiler & Kraus, 1977

Issues with using a Bulk Mixed Layer Model

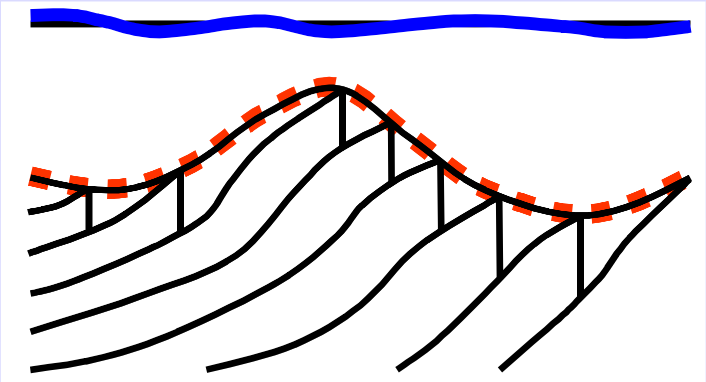
1. Momentum is not perfectly mixed; Ekman shears within the boundary layer lead to important restratification.
2. Biological tracers have mixed layer gradients that can be important (mixing timescales exceed biological timescales)
3. Spurious mixing can arise when the diagnosed boundary layer depth does not coincide with an interface (bulk mixed layers do not work well in Z-coordinate models)
4. Isopycnal coordinate models use moving interfaces to track the mixed layer depth and handle entrainment, but conservatively and non-diffusively handling detrainment gets ugly.







Isopycnic-Coord./Bulk Mixed Layer Representation



Kraus & Turner, 1967; Niiler & Kraus, 1977

A mixed layer Turbulent Kinetic Energy (TKE) budget

Bulk mixed layer entrainment is governed by a Turbulent Kinetic Energy balance:

$$w_E \frac{h}{2} g' = m^* u^{*3} + w_E m^u \|\Delta u\|^2 - n^* \frac{h}{2} \left\{ B_0 + B_{Pen} \left[\frac{2}{\lambda h} \left(1 - e^{-\lambda h}\right) - e^{-\lambda h} \right] \right\}$$

Notation:

w_E - Entrainment velocity at base of ML [m s^{-1}]

h – Bulk mixed layer thickness [m]

$g' = g \, \Delta\rho / \rho$ - Reduced gravity across base of mixed layer [m s^{-2}]

$u^* = \sqrt{\tau / \rho}$ - Friction velocity [m s^{-1}]

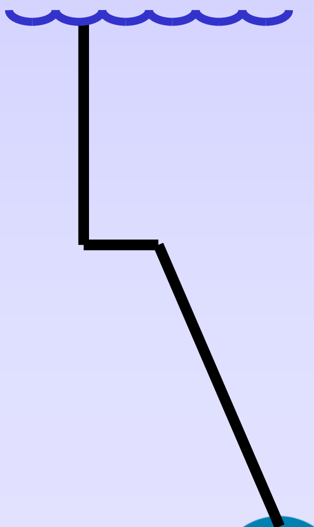
$\|\Delta u\|$ - Magnitude of velocity jump at base of mixed layer [m s^{-1}]

B_0 – Buoyancy forcing absorbed at the ocean surface [m^2 s^{-3}]

B_{Pen} – Penetrating (shortwave) buoyancy forcing [m^2 s^{-3}]

λ – Absorption rate for shortwave radiation [m^{-1}]

m^*, m^u, n^* – Order 1 dimensionless efficiencies or decay terms.



Potential Energy Change Due to Entrainment

- Physics 101: Change in potential energy due to lifting a mass M a distance Δz against gravity:

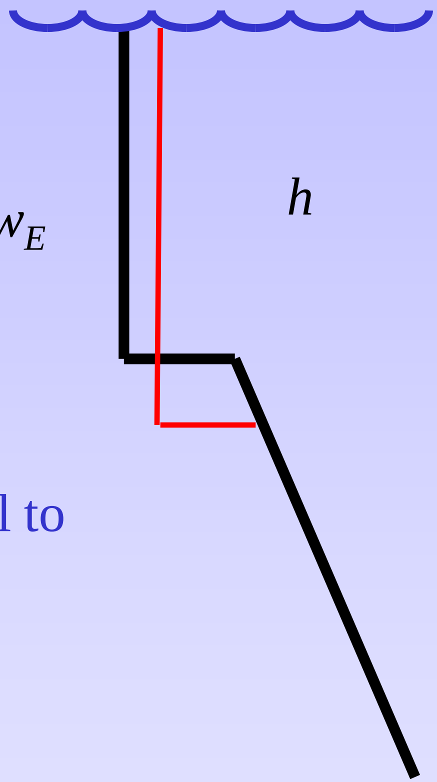
$$\Delta PE = Mg\Delta z$$

- Change in potential energy from entraining at a rate w_E for a time Δt:

$$\Delta PE = (w_E \Delta t \Delta\rho) g (h/2)$$

- Divide through by mean density and the time interval to convert to work on the column in m^3 s^{-3}.

$$Sink = w_E \frac{h}{2} g'$$



Potential Energy Change Due to Buoyancy Forcing:

- Buoyancy forcing of the ocean at the surface needs to be mixed down an average distance of *h*/2.

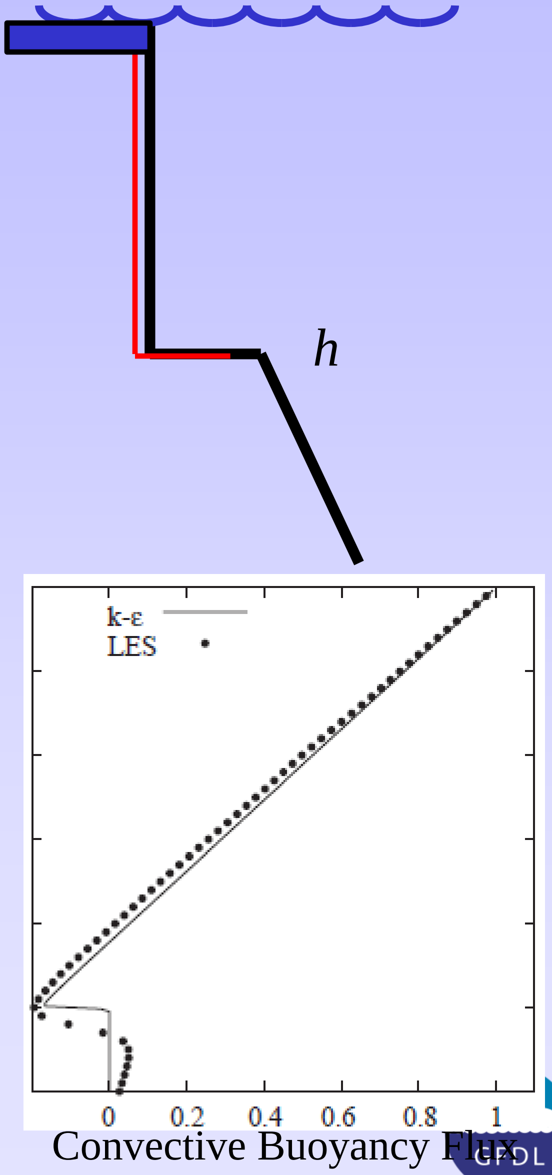
$$Sink = \tfrac{1}{2} h B_0$$

- Some shortwave heating penetrates with a distance of up to 10s of meters, so the average distance it needs to be mixing is an integral over the heating distribution:

$$Sink = n^* \frac{h}{2} B_{Pen} \left[\frac{2}{\lambda h} \left(1 - e^{-\lambda h}\right) - e^{-\lambda h} \right]$$

- Extracting buoyancy at the surface leads to free convection, but only a fraction (~20%) of the potential energy released becomes TKE.

$$Source = -0.2 \frac{h}{2} B_0$$



Potential Energy of a Column (Exact)

$$PE = \int_{-D}^{\eta} \rho g (z' + D) dz$$

$$= -\int_{p_D}^{0} (z + D) dp$$

$$= \int_0^{p_D} \int_p^{p_D} \alpha \frac{dp'}{g} dp$$

$$= \int_0^{p_D} \alpha p \frac{1}{g} dp - \left[p \int_0^{p_D} \alpha \frac{1}{g} dp \right]_{p=0}^{p_D}$$

$$\alpha \equiv \frac{1}{\rho}$$

$$\frac{dp}{dz} = -g\rho$$

$$z(p) = -D + \int_{p_D}^{p} \frac{dz}{dp} dp'$$

$$= -D - \int_{p_D}^{p} \frac{\alpha}{g} dp'$$

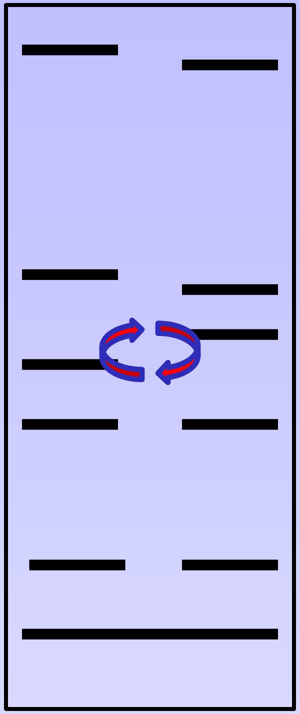
$$\int u dv = uv - \int v du$$

$$PE = \int_0^{p_D} \alpha p \frac{1}{g} dp$$

PE Change from Mixing & Conversion to TKE

Change in potential energy due to diffusivity κ_k at interface k:

$$\frac{d\dot{PE}}{d\kappa_k} = \int_0^{p_D} \frac{d\dot{\alpha}}{d\kappa_k} p \frac{1}{g} dp$$



Not all potential energy change is available for conversion to Turbulent Kinetic Energy (TKE). Energy released by contraction of the column radiates as gravity waves:

$$\frac{dT\dot{K}E}{d\kappa_k} = \left[\int_0^{p_D} \frac{d\dot{\alpha}}{d\kappa_k} p \frac{1}{g} dp - \max\left(0, p\frac{1}{g} \int_0^{p_D} \frac{d\dot{\alpha}}{d\kappa_k} dp \right) \right] \begin{pmatrix} 1 & \frac{dT\dot{K}E}{d\kappa} < 0 \\ 0.2 & \frac{dT\dot{K}E}{d\kappa} > 0 \end{pmatrix}$$

Mixing is done for conservative temperature and salinity:

$$\frac{dT\dot{K}E}{d\kappa_k} = \int_0^{p_D} \left(\frac{\partial \alpha}{\partial \theta} \frac{d\dot{\theta}}{d\kappa_k} + \frac{\partial \alpha}{\partial S} \frac{d\dot{S}}{d\kappa_k} \right) p \frac{1}{g} dp - \max\left[0, p\frac{1}{g} \int_0^{p_D} \left(\frac{\partial \alpha}{\partial \theta} \frac{d\dot{\theta}}{d\kappa_k} + \frac{\partial \alpha}{\partial S} \frac{d\dot{S}}{d\kappa_k} \right) dp \right]$$

The tridiagonal equations for the total implicit evolution of θ and S profiles can be differentiated with κ_k and integrated over the layers above using $\kappa_{k\text{-}1}$ and the expression for $\frac{dT\dot{K}E}{d\kappa_{k-1}}$

noaaresearch

GFDL

The Integrated Energetics Planetary Boundary Layer Scheme (ePBL)

- Use an implicit integrated TKE budget to constrain mixing; use the minimum of a specified TKE-dependent diffusivity or the diffusivity that can be sustained by the integrated TKE source:

$$T\dot{K}E_{3/2} = \left(m^* u^{*3} - n^* \frac{h_1}{2} \left\{ B_0 + B_{Pen,0} \left[\frac{2}{\lambda h_1} \left(1 - e^{-\lambda h_1} \right) - e^{-\lambda h_1} \right] \right\} \right) e^{-h_1 / L_{Decay}}$$

$$v^*_{k+1/2} = \sqrt[3]{T\dot{K}E_{k+1/2}} \qquad \kappa_{Tgt} = 0.4 v^*_{k+1/2} \min\left(z_{Rough} + \sum_{K=1}^{k} h_K , \frac{v^*_{k+1/2}}{|f|} \right)$$

$$T\dot{K}E_{Tgt} = T\dot{K}E_{k+1/2} + \Delta T\dot{K}E\left(\kappa_{Tgt}\right)$$

$$\kappa_{k+1/2} = \begin{cases} \kappa_{Tgt} & T\dot{K}E_{Tgt} \geq 0 \\ \Delta T\dot{K}E^{-1}\left(-T\dot{K}E_{k+1/2}\right) & T\dot{K}E_{Tgt} \geq 0 \end{cases}$$

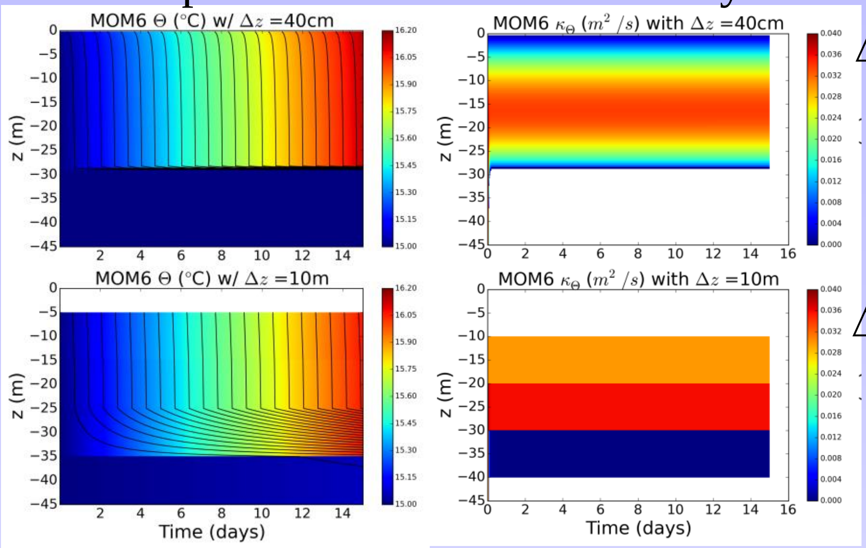
$$T\dot{K}E_{k+3/2} = \left[T\dot{K}E_{k+1/2} + \Delta T\dot{K}E\left(\kappa_{k+1/2}\right) \right] \exp^{-h_k / L_{decay}}$$

- The APE change calculation needs to be fully implicit.
- Specifying an infinite target diffusivity replicates a bulk mixed layer model.
- A mean-kinetic energy to TKE term could also be included.

Idealized 1-D Case with Skin Heating and Wind Forcing

ePBL

Temperature | Diffusivity

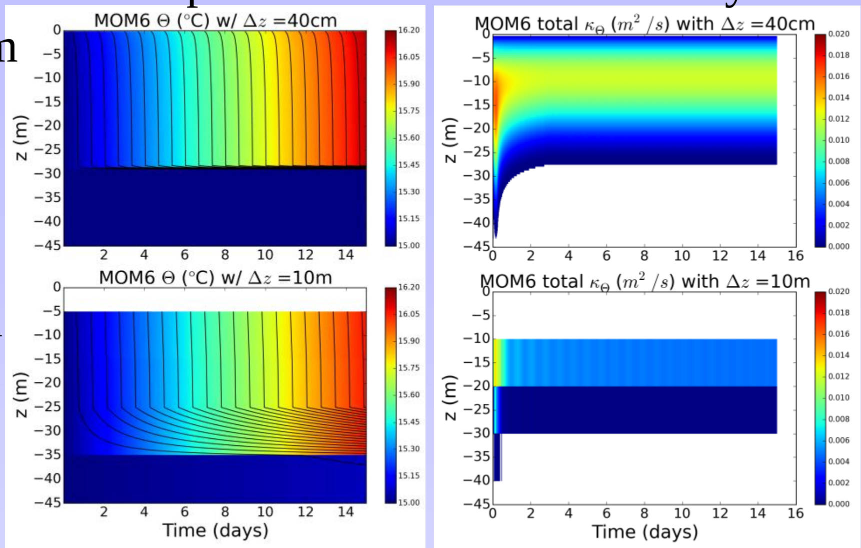


Δz=0.4m

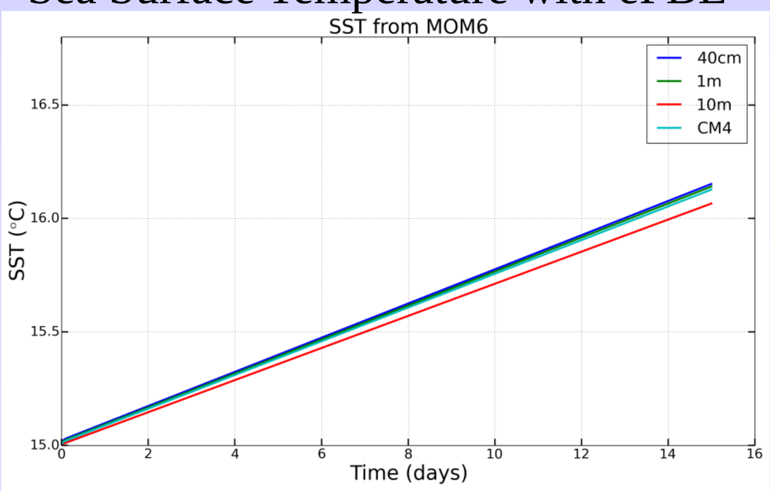
Δz=10m

KPP

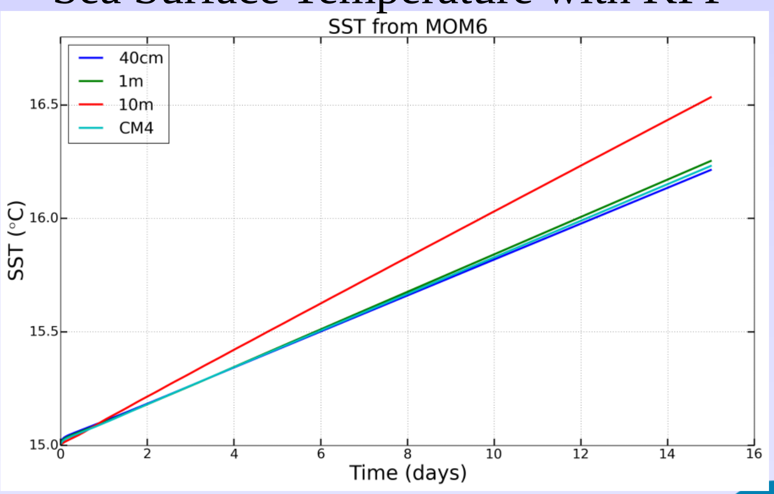
Temperature | Diffusivity



Sea Surface Temperature with ePBL

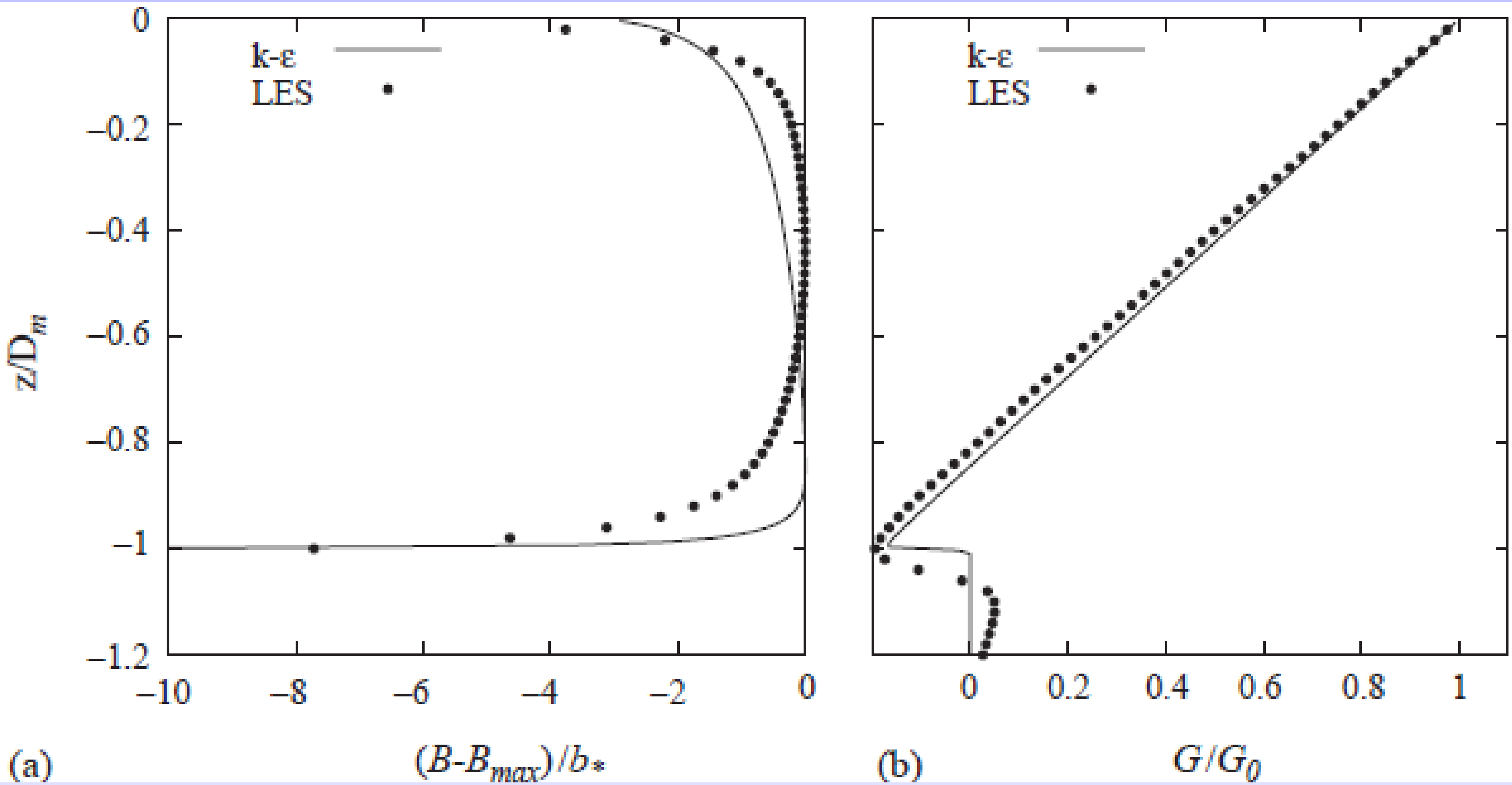


Sea Surface Temperature with KPP



k-ε Simulation of a Convecting Boundary Layer

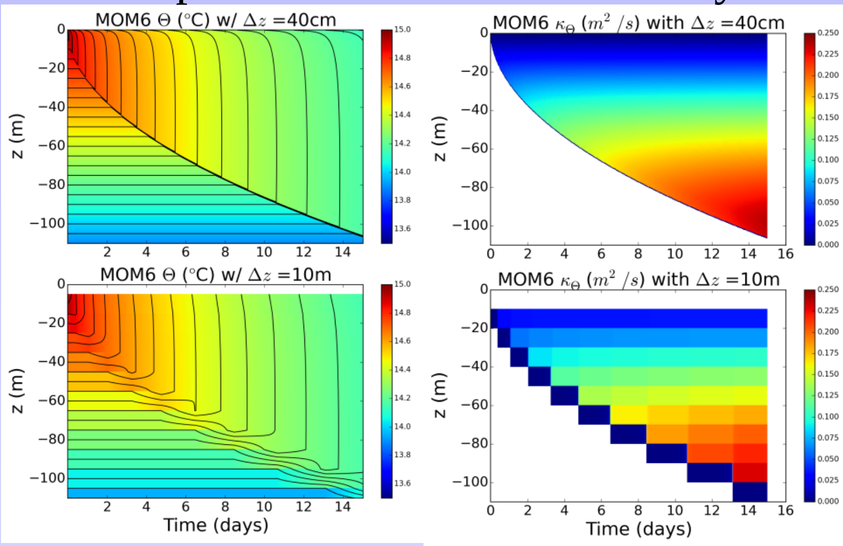
Buoyancy Profile | Upward Buoyancy Flux



Idealized 1-D Case with Surface Cooling (no wind)

ePBL

Temperature / Diffusivity

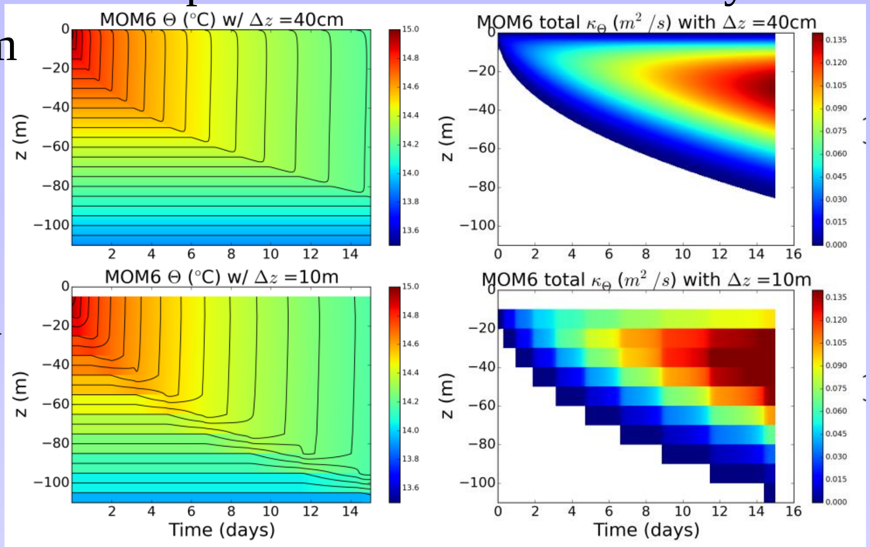


Δz=0.4m

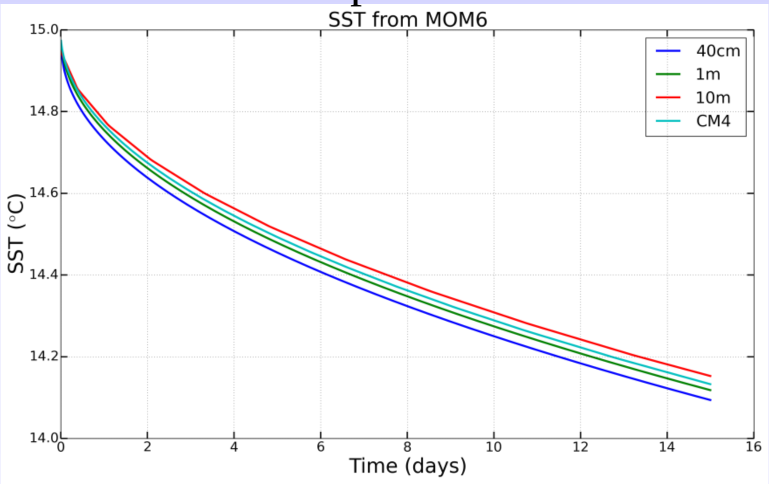
Δz=10m

KPP

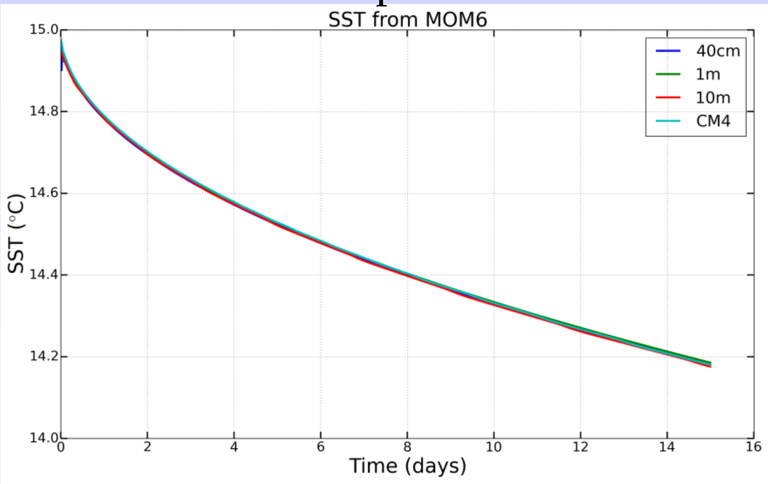
Temperature / Diffusivity



Sea Surface Temperature with ePBL



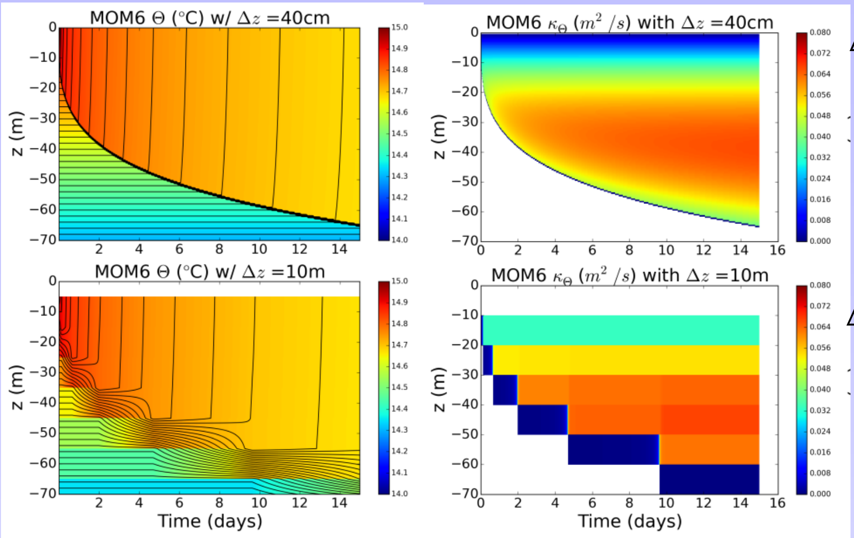
Sea Surface Temperature with KPP



Idealized 1-D Case with Wind-Forcing (no Heating)

ePBL

Temperature | Diffusivity

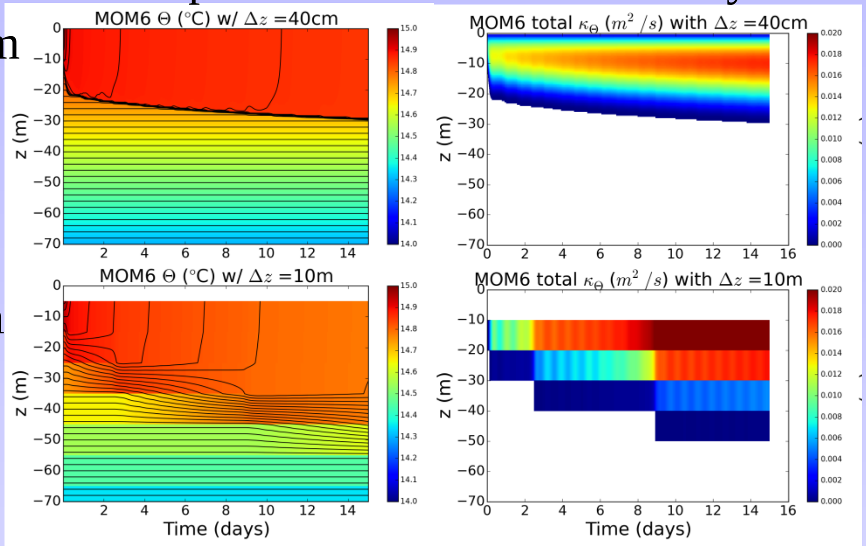


Δz=0.4m

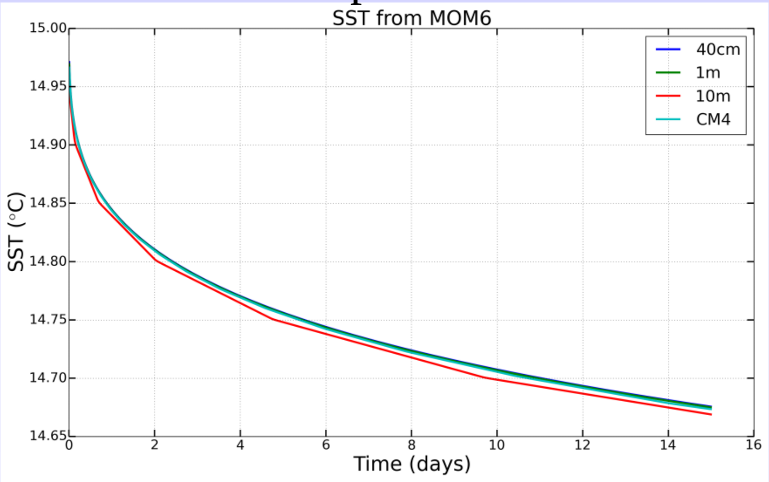
Δz=10m

KPP

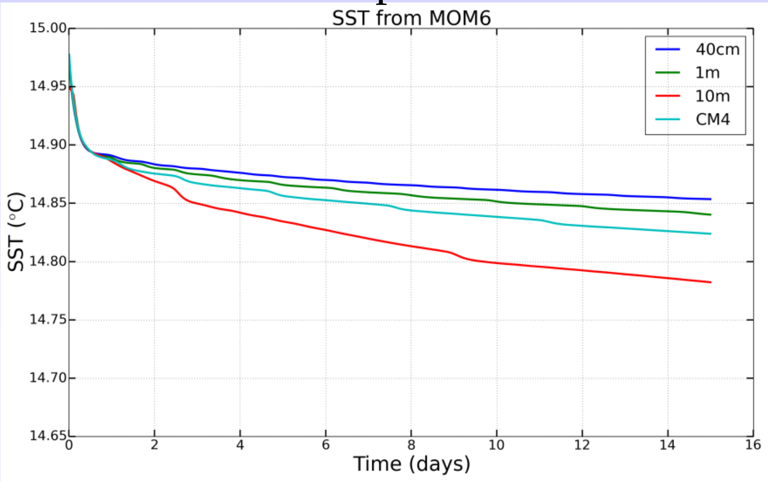
Temperature | Diffusivity



Sea Surface Temperature with ePBL

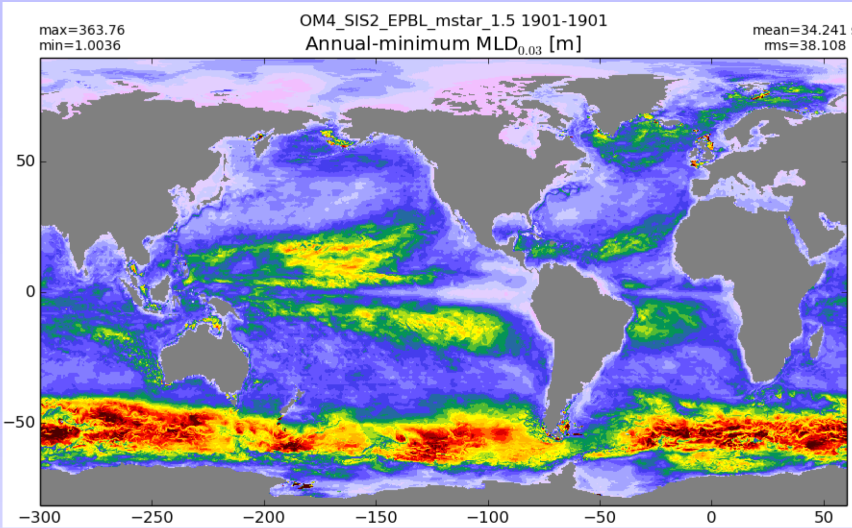


Sea Surface Temperature with KPP

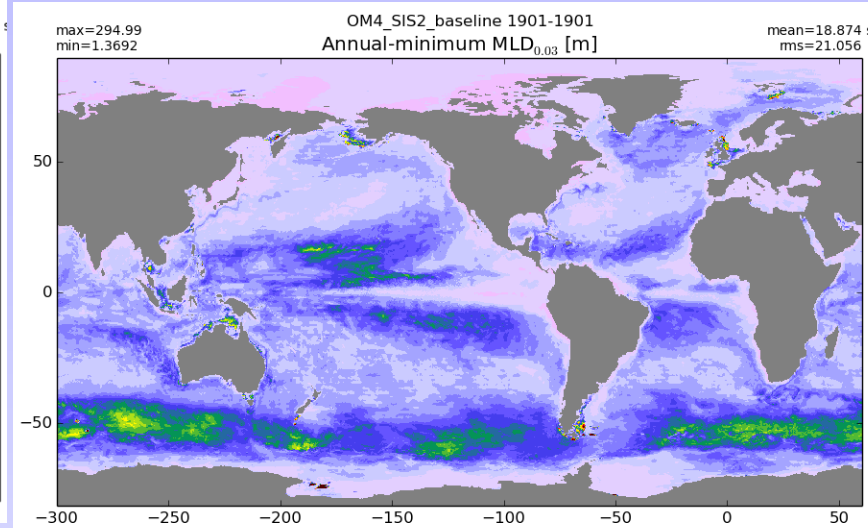


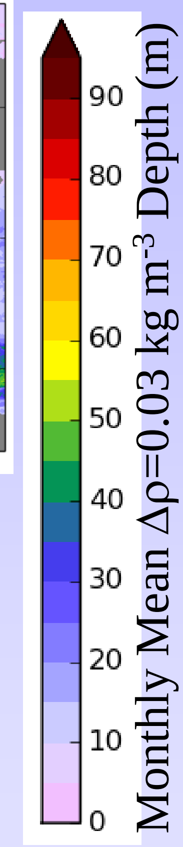
Minimum Monthly-Mean Mixed Layer Depths

ePBL

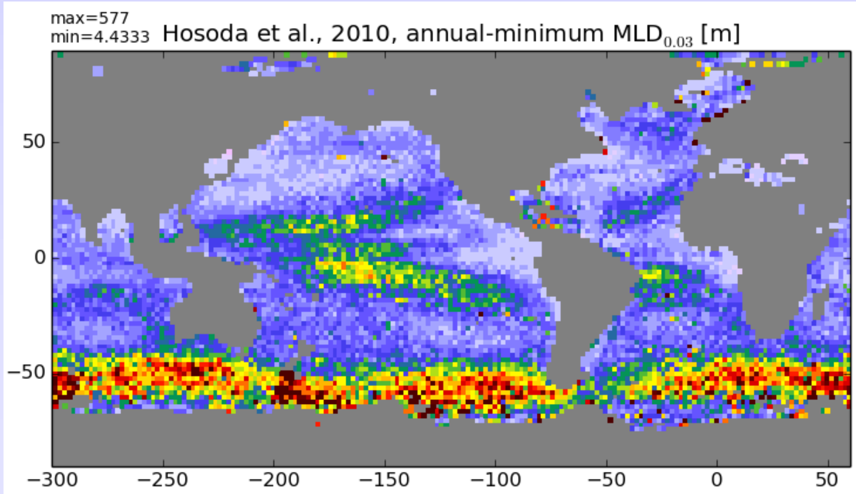


KPP





Observed

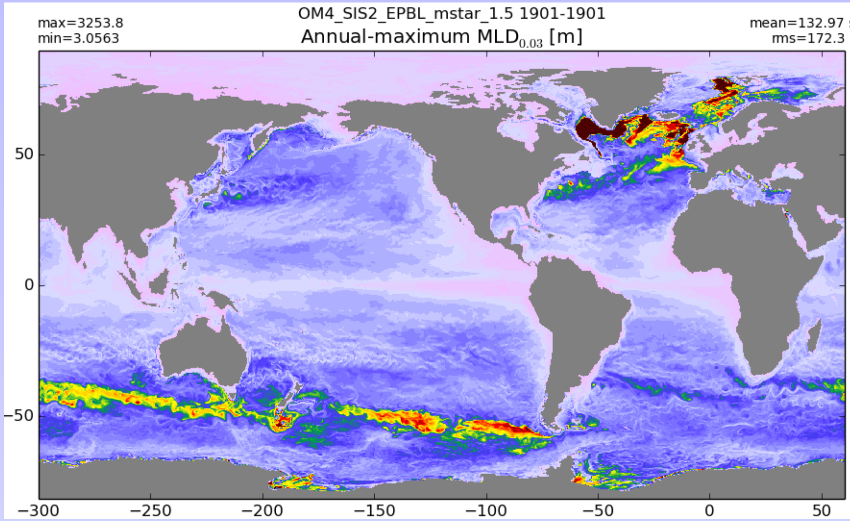


noaaresearch

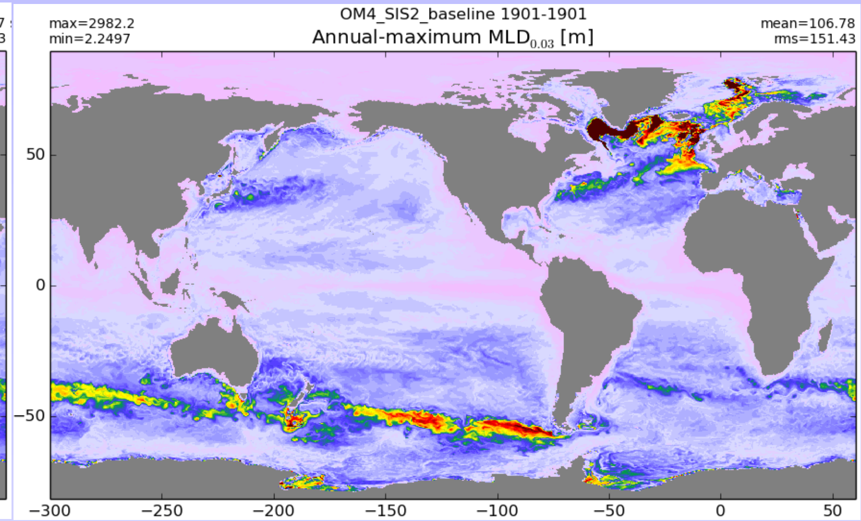
GFDL

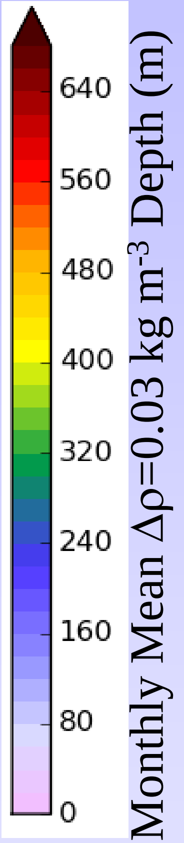
Maximum Monthly-Mean Mixed Layer Depths

ePBL

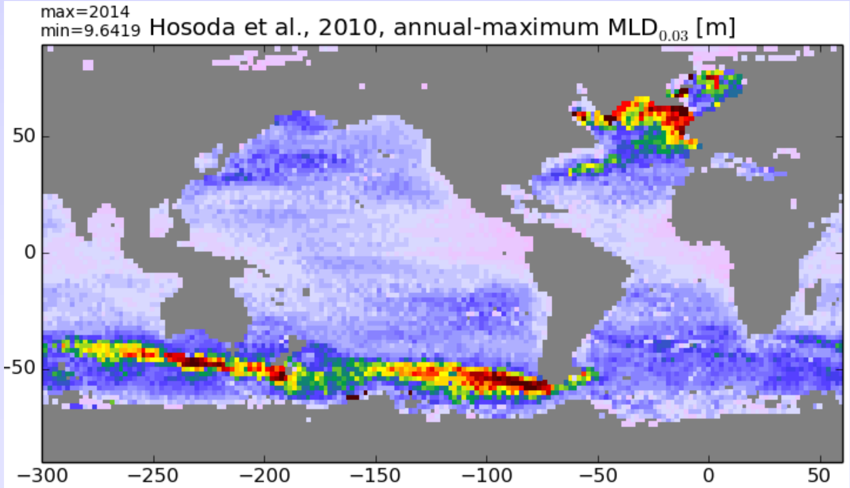


KPP



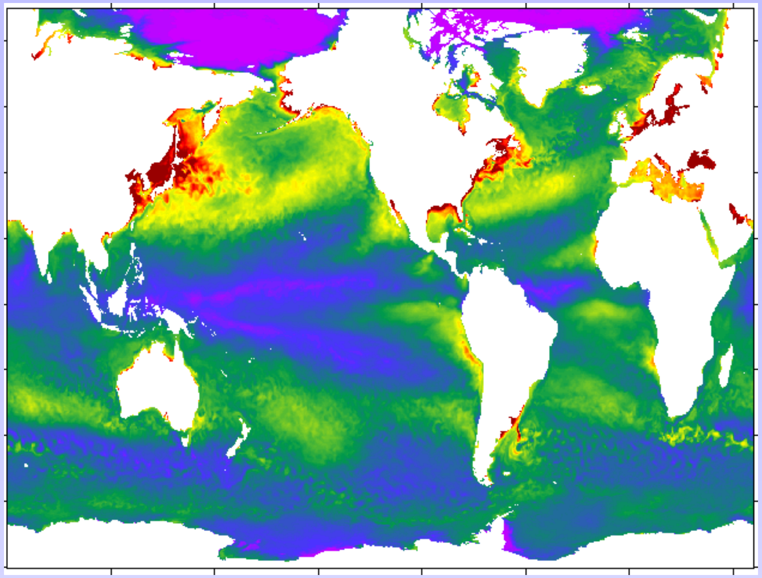


Observed

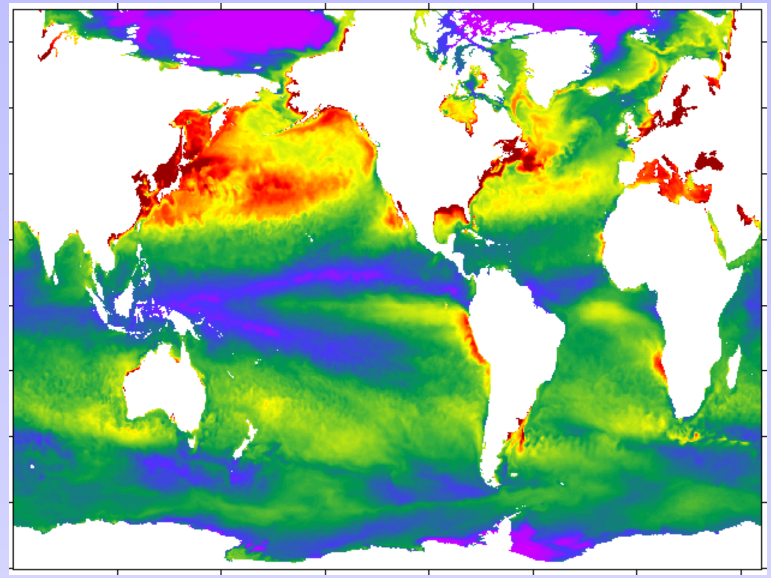


Magnitude of Seasonal Cycle: Range of Monthly Mean Sea Surface Temperatures

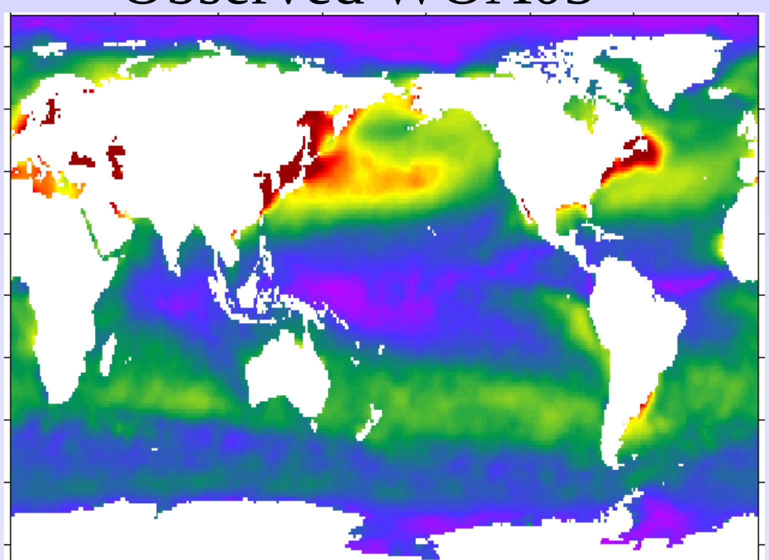
ePBL

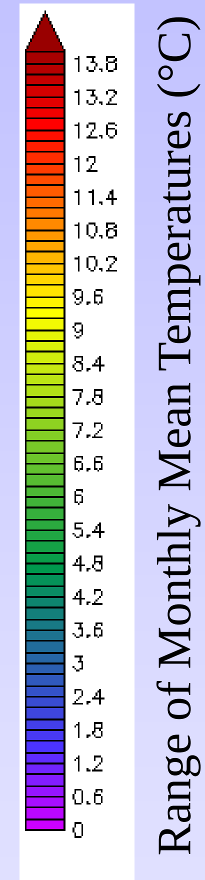


KPP



Observed WOA05





Summary

Energetics constrain boundary layer mixing.

The new integrated energetics planetary boundary layer model (ePBL) seems to work very robustly in idealized and realistic tests with finite diffusivities.

ePBL captures the physical content of bulk mixed layer ideas, but works well for any coordinate system.

In tests with GFDL's new coupled model, OM4, ePBL is numerically more stable and less resolution dependent than KPP.