Eddy-induced transport in layered models

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Outline

- Parameterized eddy-induced transport.
- Gent-McWilliams parameterization.
- Comparison with the MICOM/HYCOM approach.
- Results from idealized experiments.
- Results from realistic global model configurations.

Parameterized eddy-induced transport

- The parameterization of mesoscale eddies is often realized by tracer diffusion along neutral surfaces in combination with an eddy-induced transport that adiabatically tend to reduce available potential energy.
- The latter is usually achieved with some variant of the Gent-McWilliams (GM) parameterization.
- In MICOM/HYCOM, eddy-induced transport is parameterized by layer interface smoothing that is similar to GM with some exceptions:
    - GM is trying the flatten neutral surfaces, not isopycnal surfaces.
    - Interface smoothing is moving the model state towards a state with interfaces of constant pressure (if a Laplacian operator is used) that is not necessarily a state of no remaining available potential energy.
    - Near the surface, interface smoothing is profoundly different from GM when a non-isopycnic bulk surface mixed layer (ML) or hybrid layers with a pressure vertical coordinate is present.

GM parameterization

The eddy-induced velocity can be expressed as the curl of a stream function

$$\mathbf{v}^* = \nabla \times \mathbf{\Psi}, \quad \mathbf{\Psi} = \mathbf{\Upsilon} \times \mathbf{k}.$$

Here $\mathbf{\Upsilon}$ is the parameterized eddy-induced transport. Horizontal and vertical eddy-induced velocities becomes

$$\mathbf{u}^* = \frac{\partial \mathbf{\Upsilon}}{\partial z}, \quad w^* = -\nabla_z \cdot \mathbf{\Upsilon}.$$

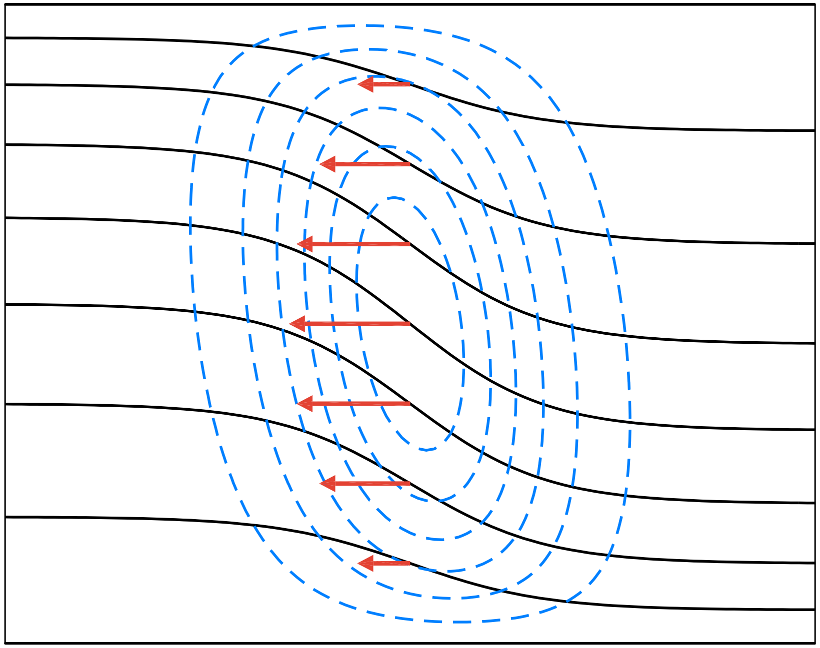
Following Gent and McWilliams (1990); Gent et al. (1995), the eddy-induced transport takes the form

$$\mathbf{\Upsilon} = -\kappa \mathbf{S},$$

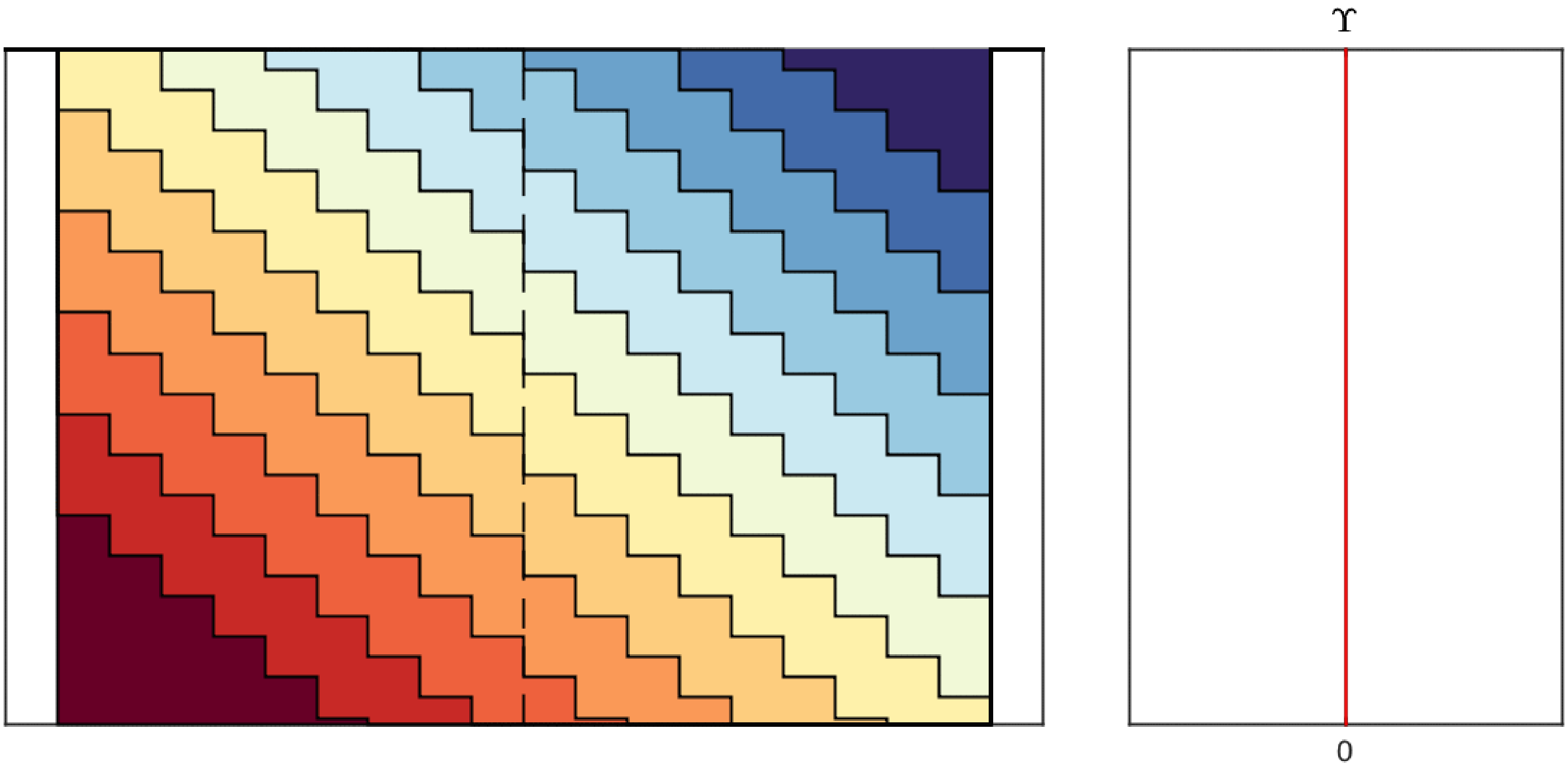
where κ is an eddy diffusivity and

$$\mathbf{S} = -\frac{\nabla_z \rho|_p}{\left.\frac{\partial \rho}{\partial z}\right|_p},$$

is the slope vector of a local neutral surface.



Eddy-induced transport by interface smoothing.



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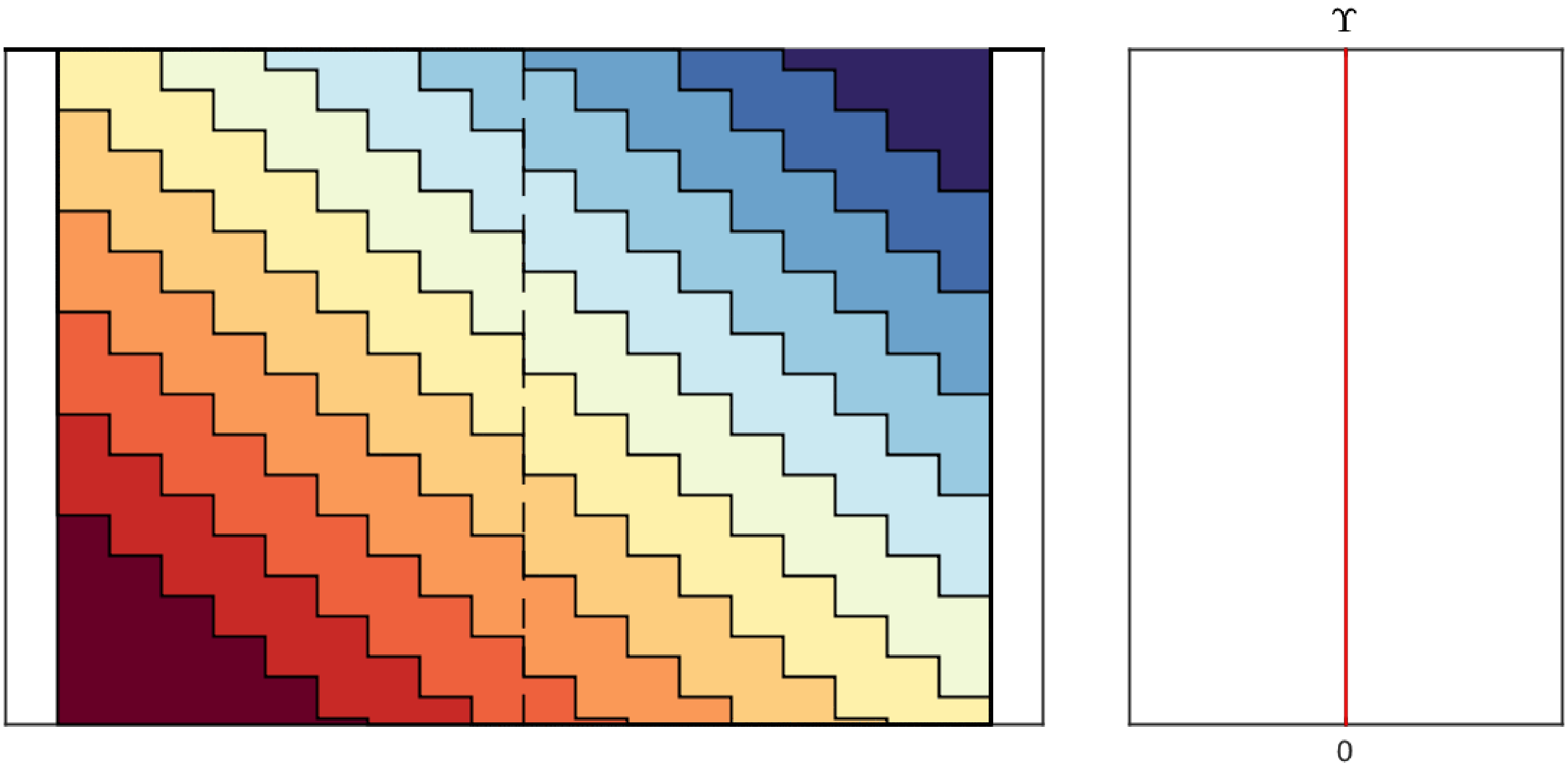


GM with a generalized vertical coordinate

Let s be a generalized vertical coordinate. Using

$$\nabla_z = \nabla_s - \nabla_s z \frac{\partial}{\partial z}, \quad \phi = gz, \quad N^2 = -\frac{g}{\rho} \left.\frac{\partial \rho}{\partial z}\right|_p ,$$

the slope vector can be expressed by gradients on s-surfaces

$$\mathbf{S} = -\frac{\nabla_z \rho|_p}{\left.\frac{\partial \rho}{\partial z}\right|_p} = \frac{g}{\rho N^2} \nabla_s \rho|_p + \frac{1}{g} \nabla_s \phi.$$

The layer pressure thickness conservation equation for a layered model is

$$\frac{\partial}{\partial t} \left.\left(\frac{\partial p}{\partial s}\right)\right|_s + \nabla_s \cdot \left(\frac{\partial p}{\partial s}\mathbf{u}\right) + \frac{\partial}{\partial s}\left(\frac{\partial p}{\partial s}\dot{s}\right) = 0.$$

The eddy-induced transport of layer pressure thickness becomes

$$\frac{\partial p}{\partial s}\mathbf{u}^* = -g\rho \frac{\partial z}{\partial s}\frac{\partial \boldsymbol{\Upsilon}}{\partial z} = -g\rho \frac{\partial \boldsymbol{\Upsilon}}{\partial s}.$$

Note that w^* is handled implicitly.

Regularizing N^2

Direct estimation of the slope vector is problematic when N^2 is small. This is particularly an issue in the surface ML.

Our current strategy is to smooth N^2 by solving the following equation implicitly for $\bar{N}^2$:

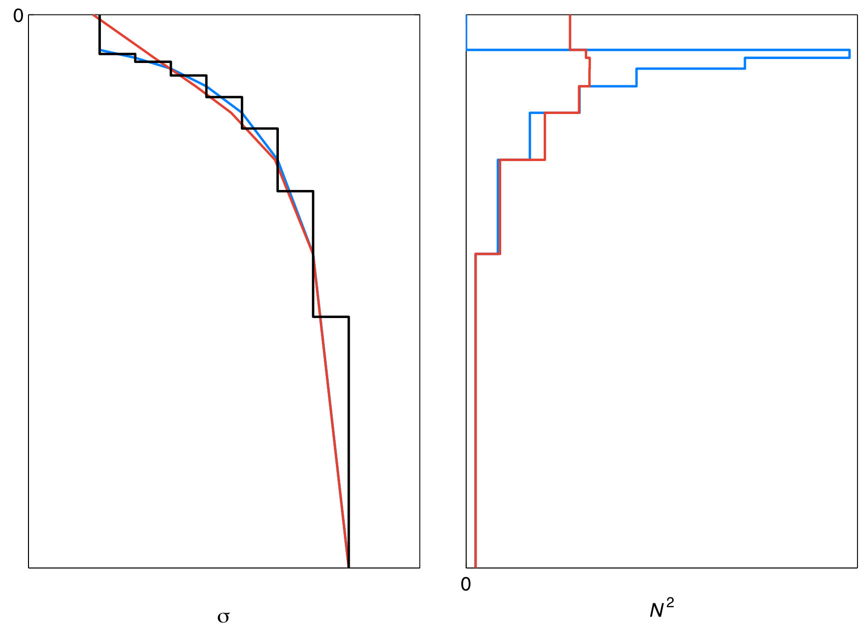
$$\bar{N}^2 - N^2 = \frac{\partial}{\partial p}\left(L^2 \frac{\partial \bar{N}^2}{\partial p}\right)$$

The smoothing length scale L is set to 2 times the ML thickness at the ML base, then approaching 10 dbar at depth with an e-folding length scale of 2 times the ML depth.

We are also looking into the approach by Ferrari et al. (2010):

$$\left(c^2 \frac{\partial^2}{\partial z^2} - N^2\right)\widetilde{\boldsymbol{\Upsilon}} = \frac{g\kappa}{\rho_0}\nabla_z \rho,$$

$$\widetilde{\boldsymbol{\Upsilon}}(\eta) = \widetilde{\boldsymbol{\Upsilon}}(-H) = 0.$$



Comparison with the MICOM/HYCOM approach

Eddy-induced transport in MICOM/HYCOM has been implemented by interface pressure smoothing.

The interface smoothing has been carried out with either a Laplacian operator

$$\boldsymbol{\Upsilon} = \frac{\kappa}{g\rho_0}\nabla_s p \approx -\kappa\left(\cancel{\frac{g}{\rho N^2}\nabla_s\rho\big|_p} + \frac{1}{g}\nabla_s\phi\right),$$

or a biharmonic operator

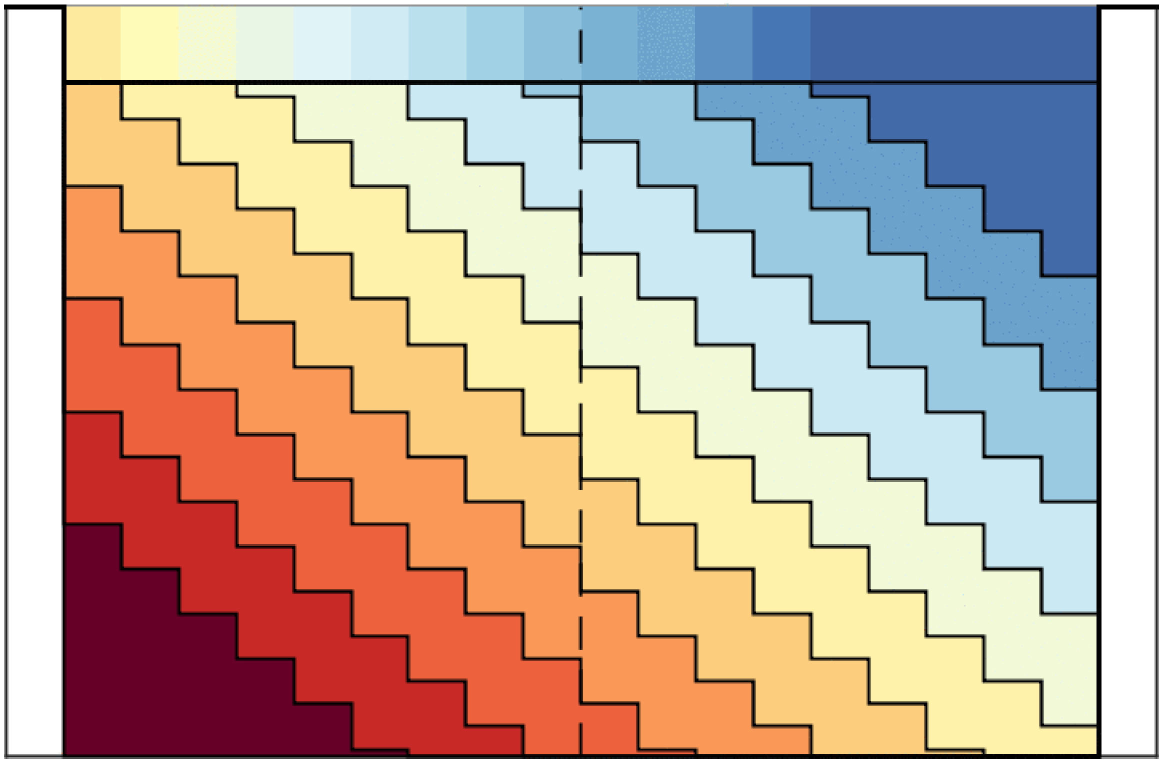
$$\boldsymbol{\Upsilon} = -\frac{\lambda}{g\rho_0}\nabla_s\nabla_s^2 p.$$

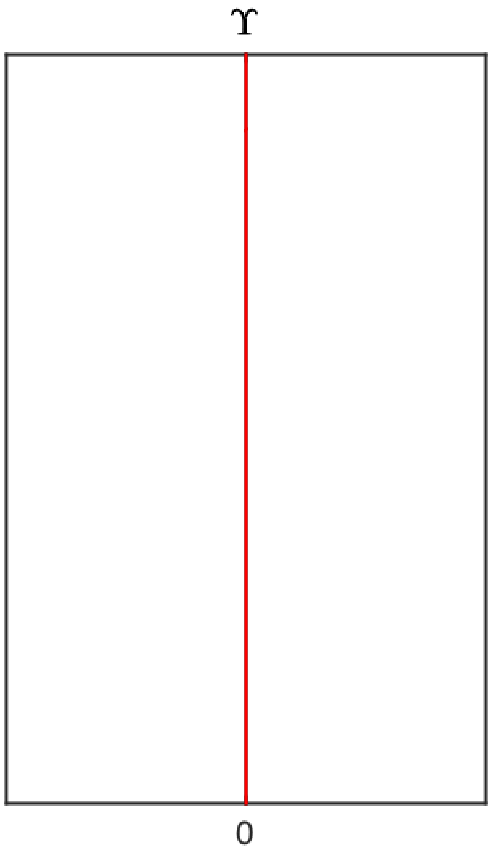
Using an Laplacian operator is a reasonable approximation to GM when applied to interfaces of isopycnic layers, while a biharmonic operator is *inconsistent* with GM.

Depending on choice of boundary conditions, the biharmonic operator does not necessarily reduce available potential energy.

Isopycnic interfaces with constant pressures does not guaranty vanishing pressure gradient forces, thus interface pressure smoothing will not necessarily always remove available potential energy.

Eddy-induced transport by interface smoothing. A variable density bulk ML is present. Hydrostatic instabilities are removed and the ML depth is restored to the constant initial depth.





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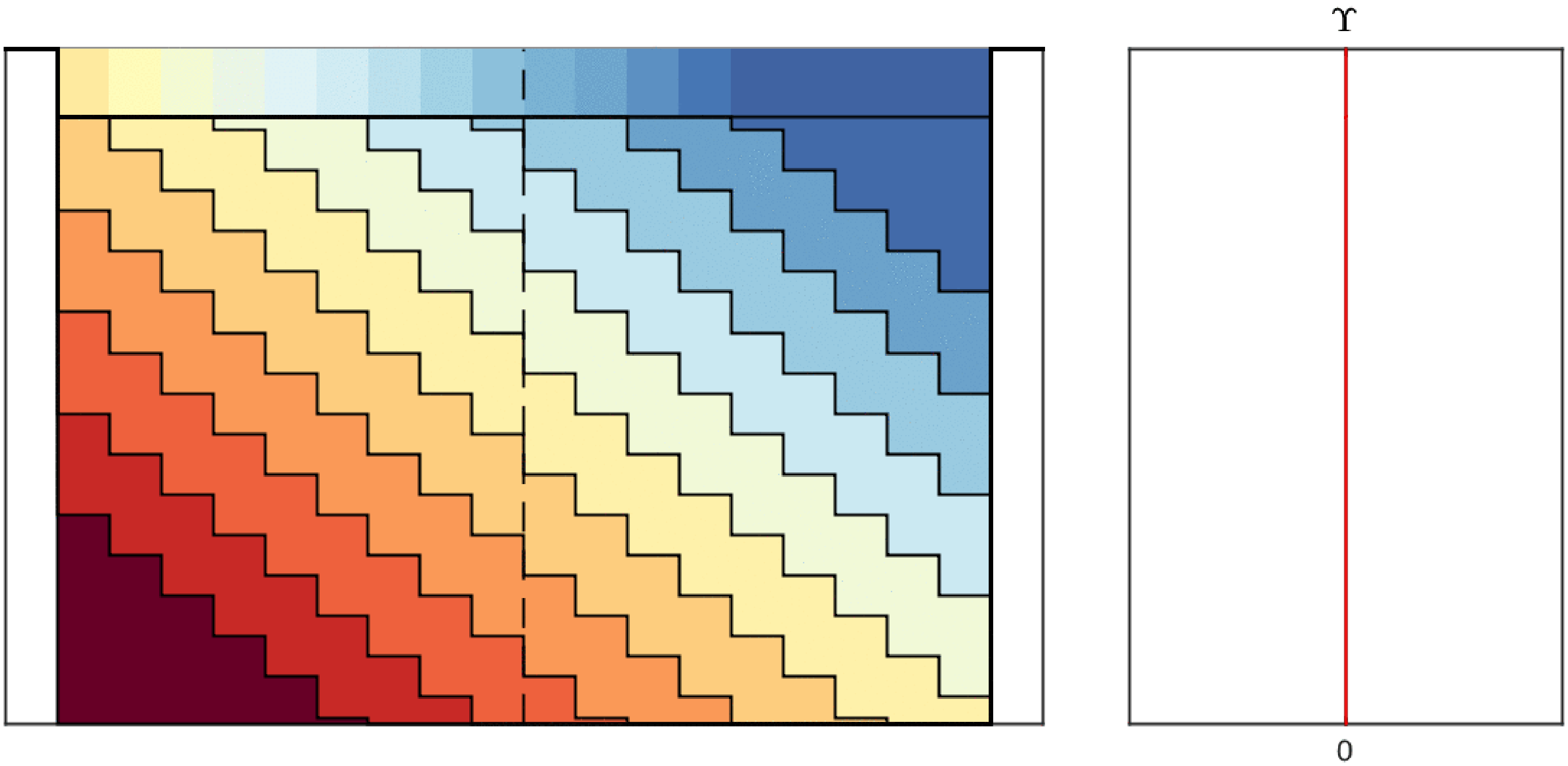
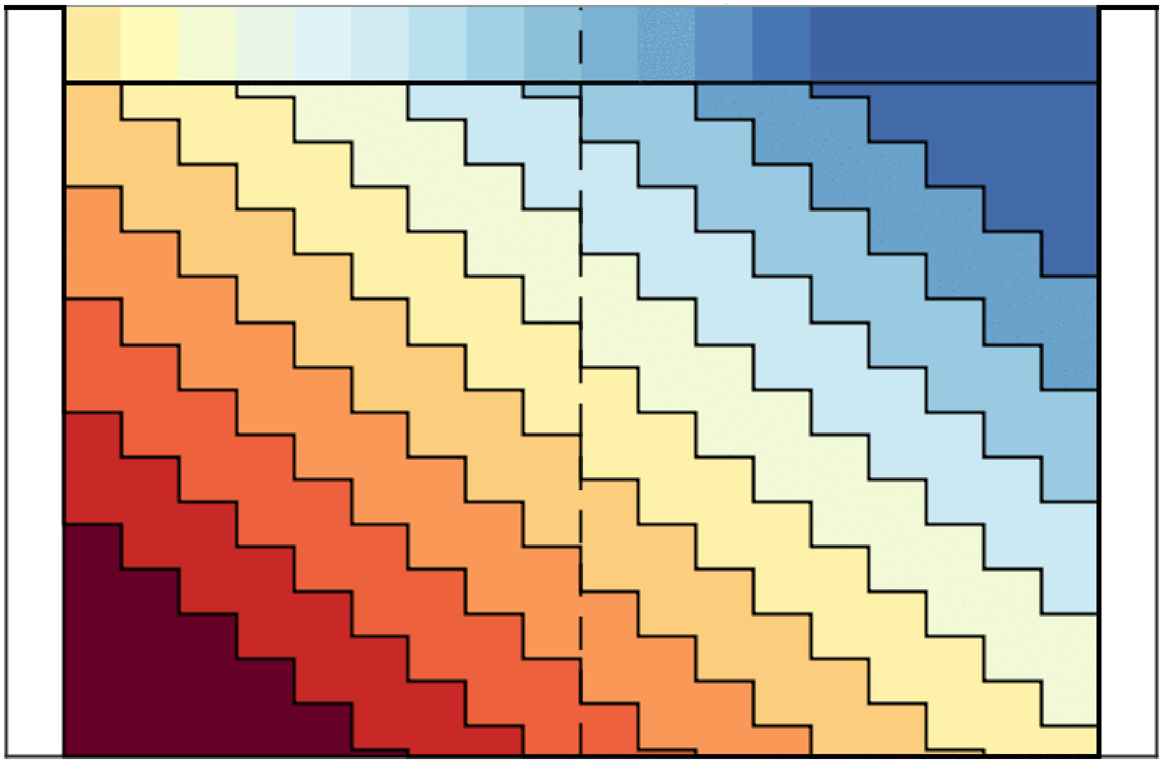
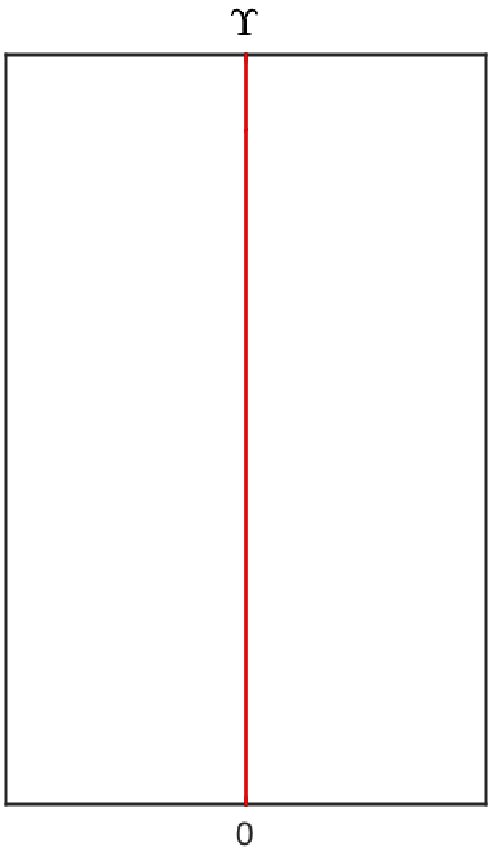


Eddy-induced transport following the GM parameterization. A variable density bulk ML is present. No removal of hydrostatic instabilities and restoring of ML depth.





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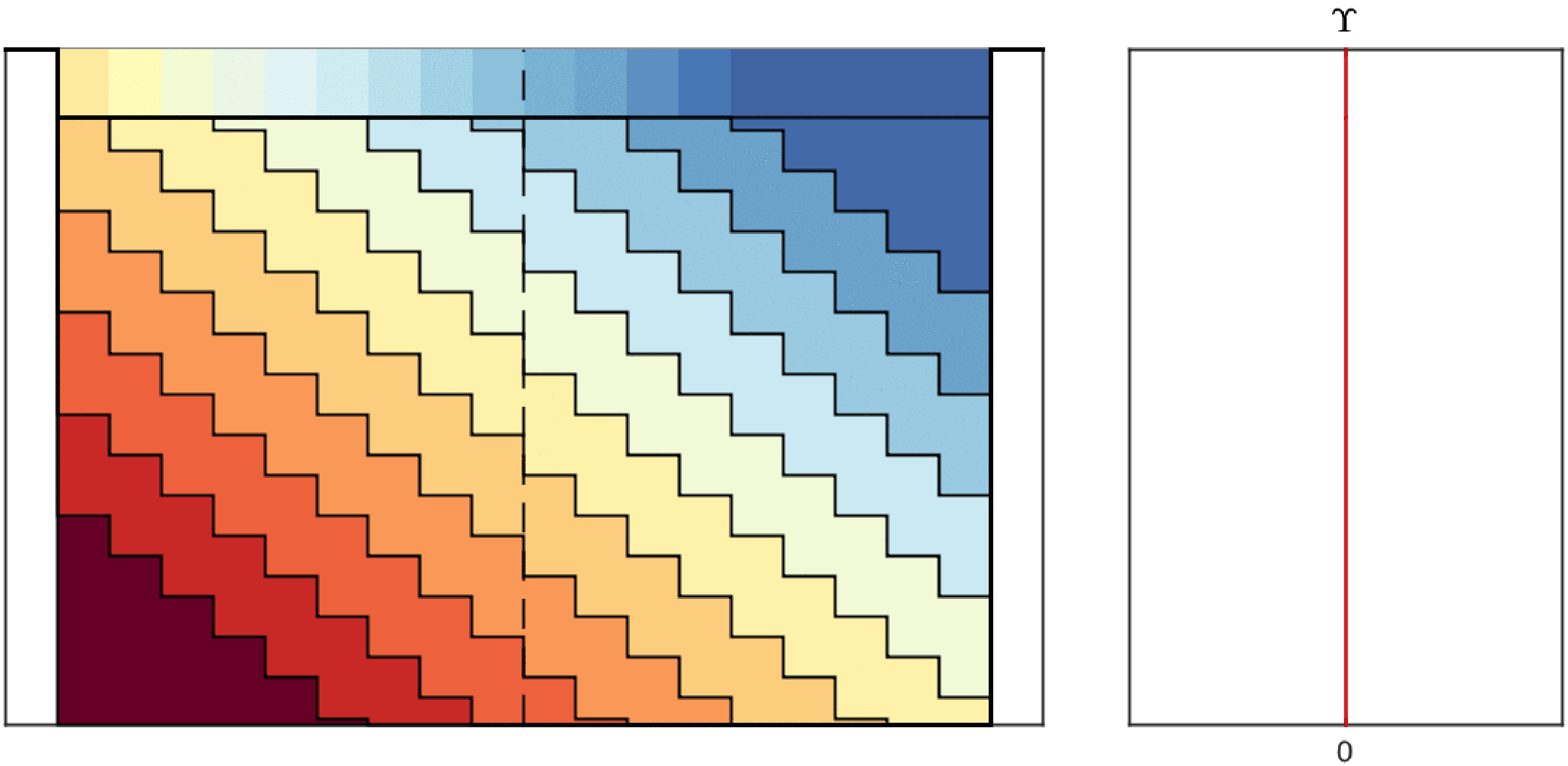
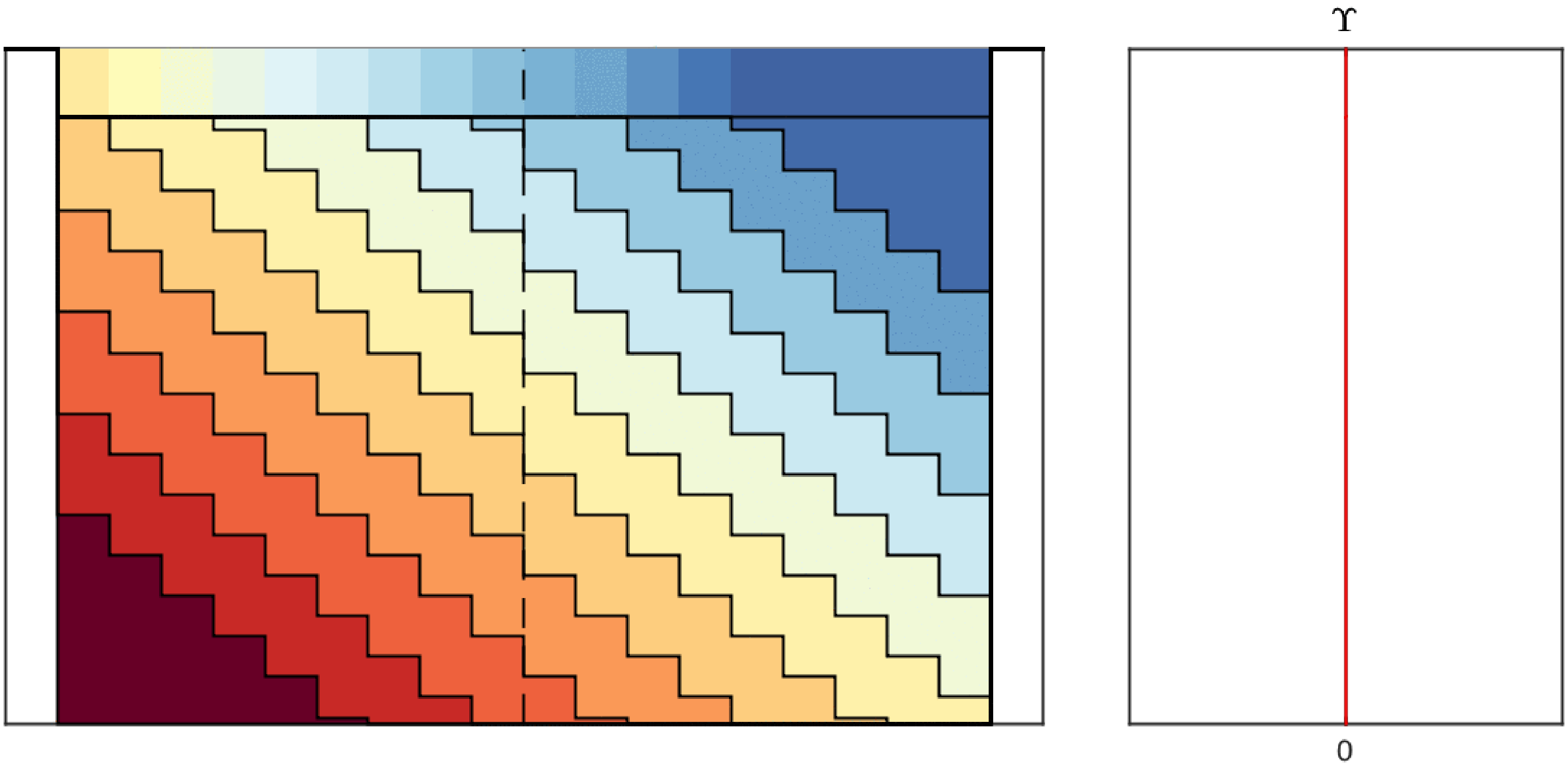


Eddy-induced transport following the GM parameterization. A variable density bulk ML is present. Hydrostatic instabilities are removed and the ML depth is restored.



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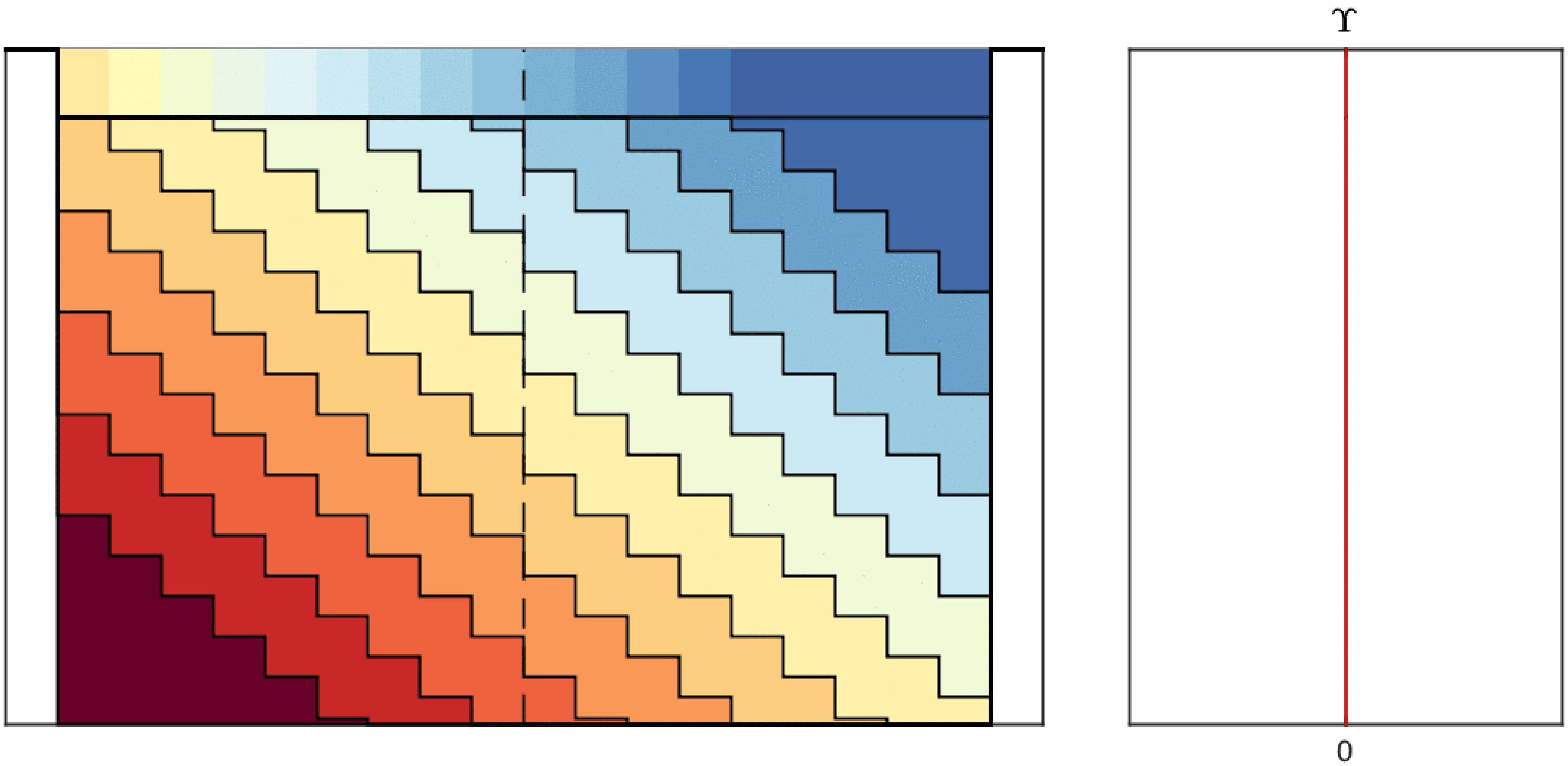


Results from realistic global model configurations

Experiments:

- **CORE2**: NorESM 1° MICOM/CICE forced with CORE2 interannual atmospheric fields and runoff. Comparing years 121-150 after start from climatological initial conditions.
- **Coupled**: NorESM 2° CAM4/CLM4, 1° MICOM/CICE, pre-industrial configuration. Comparing years 71-100 after start from climatological initial conditions.

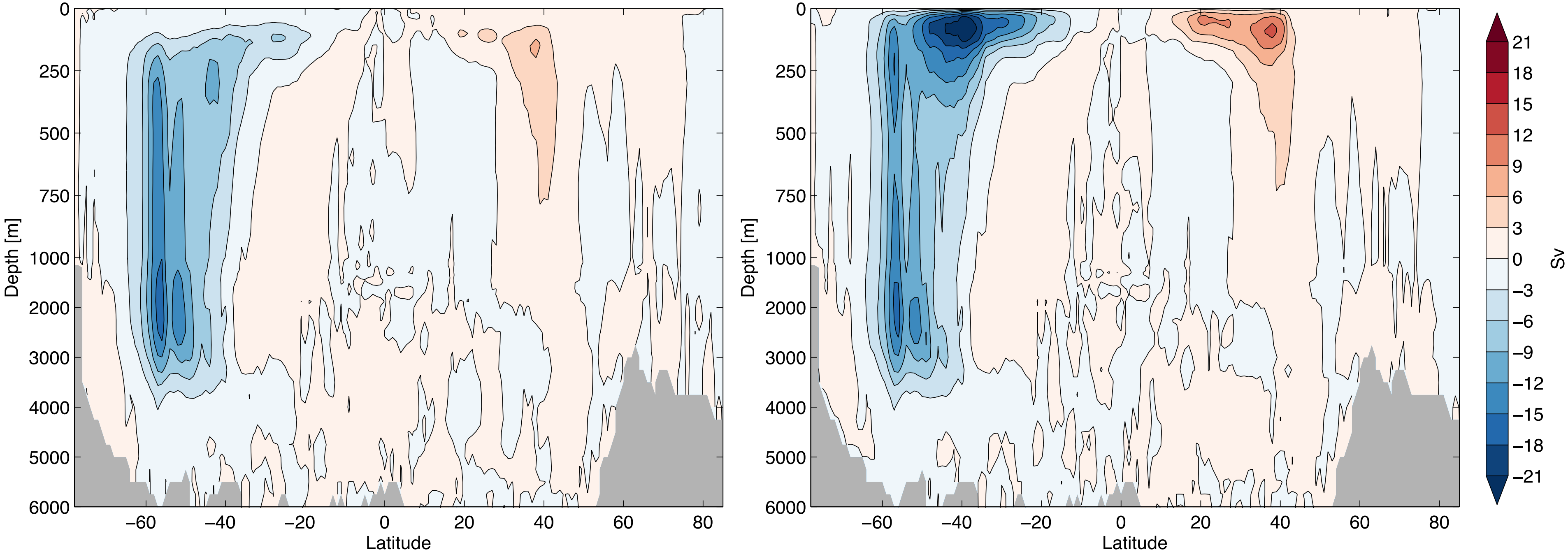
Old GM refers to Laplacian smoothing of interface pressure. The interface of the bulk ML base is not smoothed.

New GM refers to applying the eddy-induced transport according to the GM parameterization with the regularized $\bar{N}^2$.

The parameterization of eddy diffusivity follows the diagnostic version of the eddy closure of Eden and Greatbatch (2008) as implemented by Eden et al. (2009).

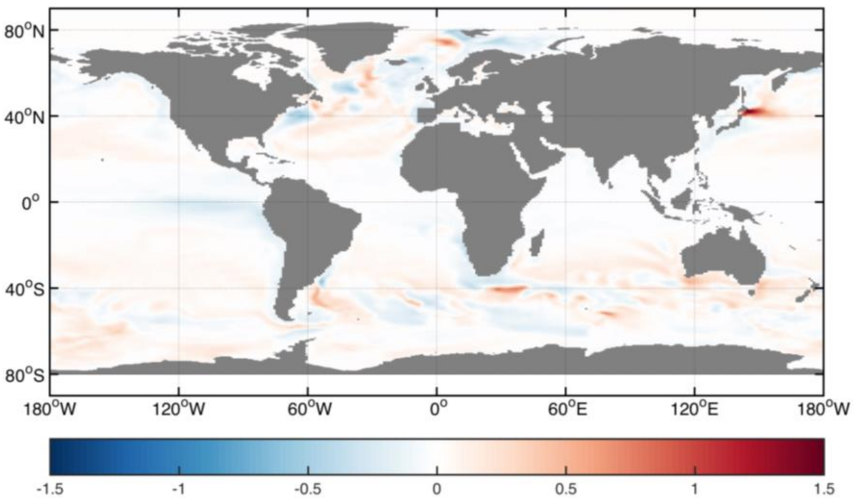
CORE2:

Overturning stream function due to eddy-induced transport (Sv).

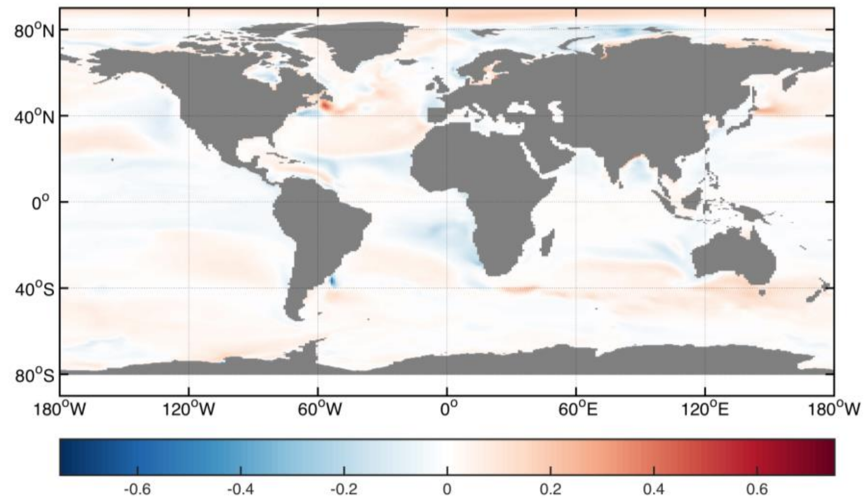


Old GM New GM

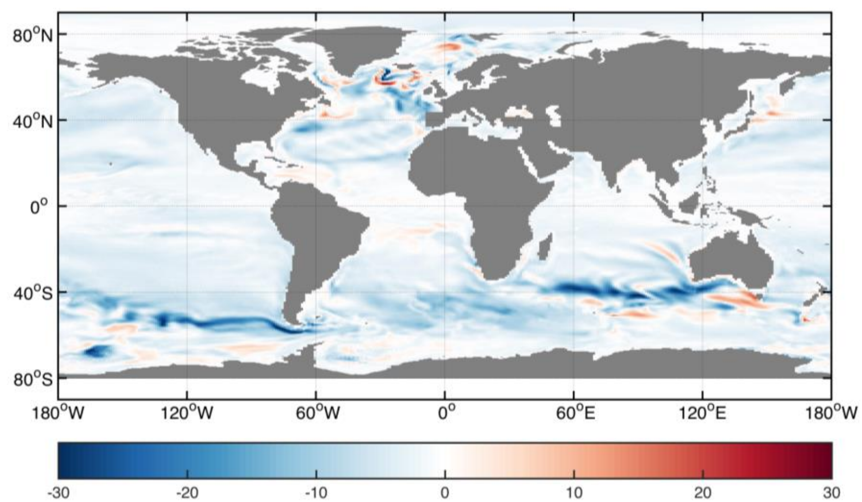
CORE2: New GM – Old GM



SST (K)



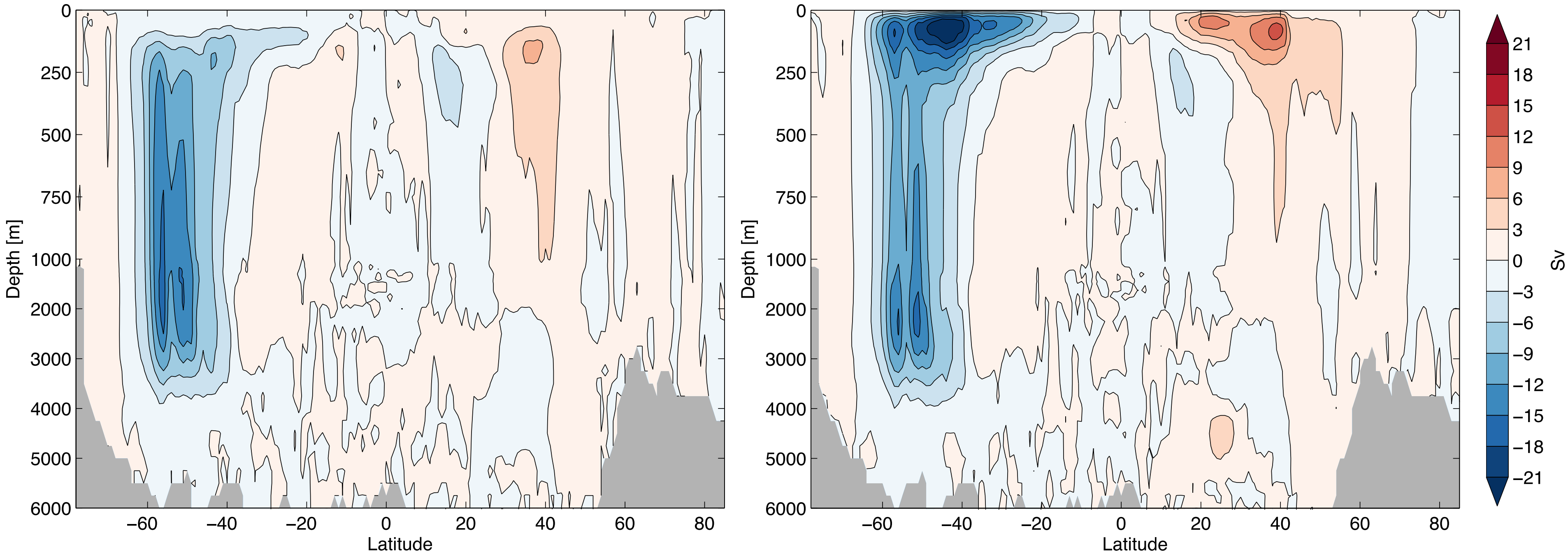
SSS (g kg-1)



MLD (m)

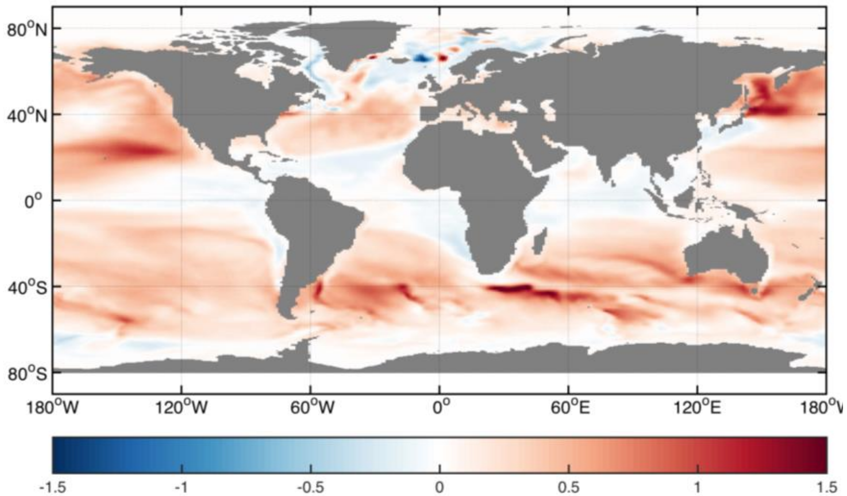
Coupled:

Overturning stream function due to eddy-induced transport (Sv).

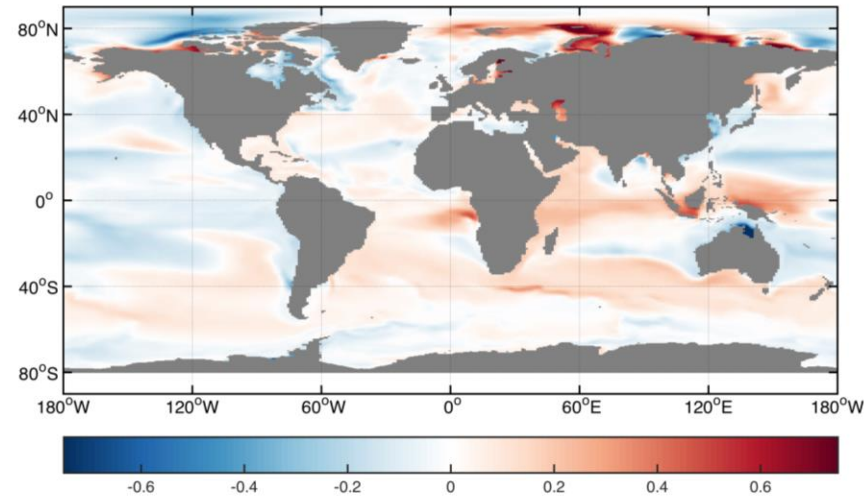


Old GM | New GM

Coupled: New GM – Old GM



SST (K)



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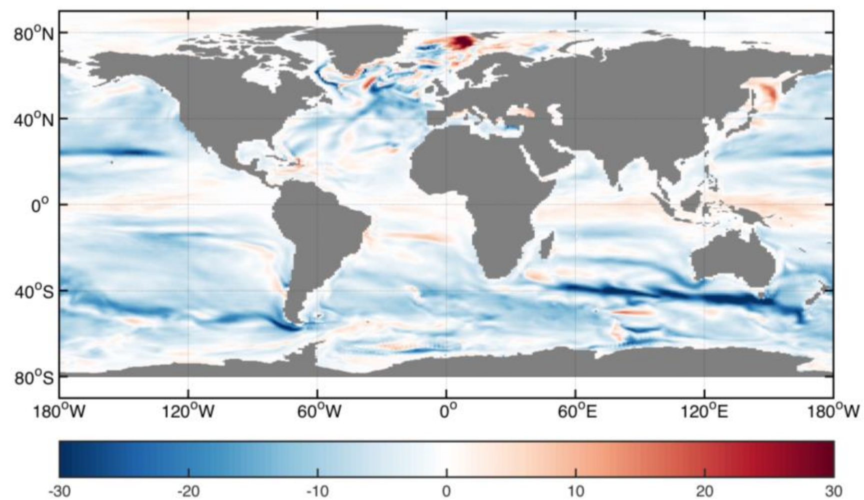
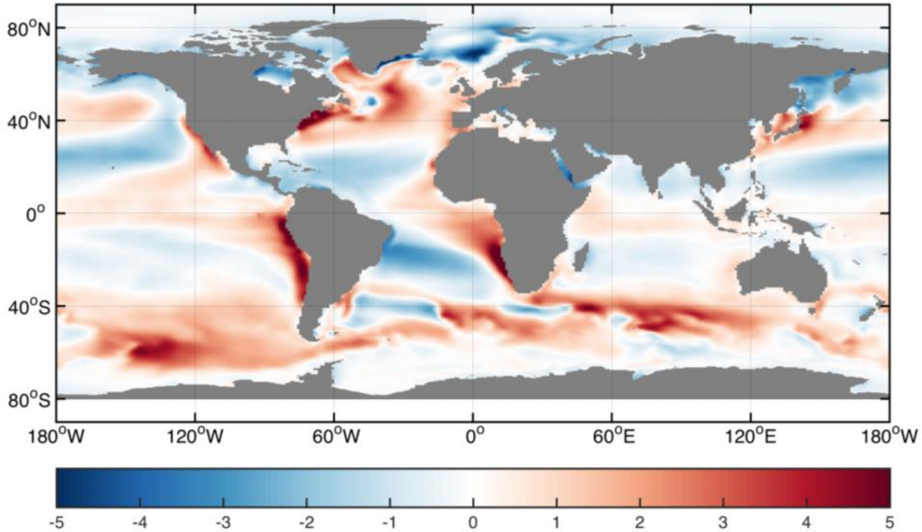
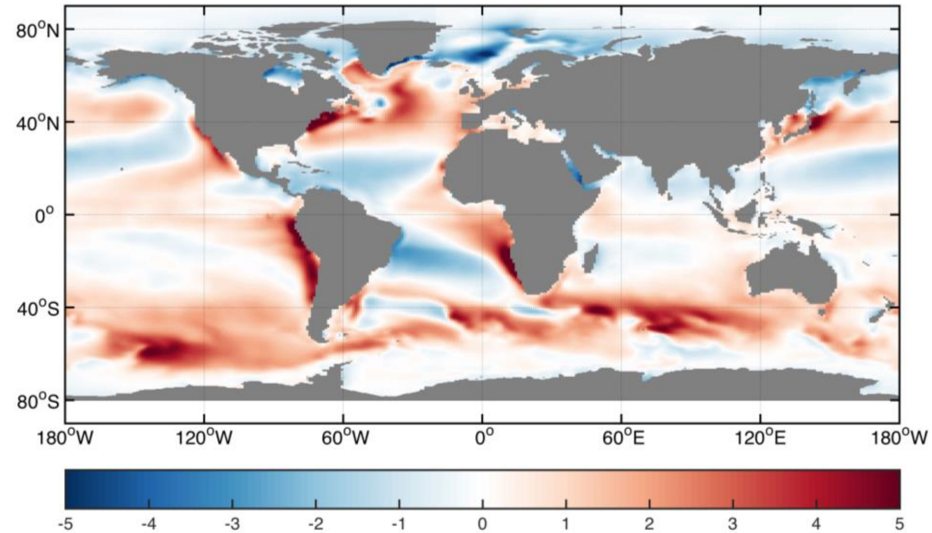


MLD (m)

Coupled: Model SST – WOA09

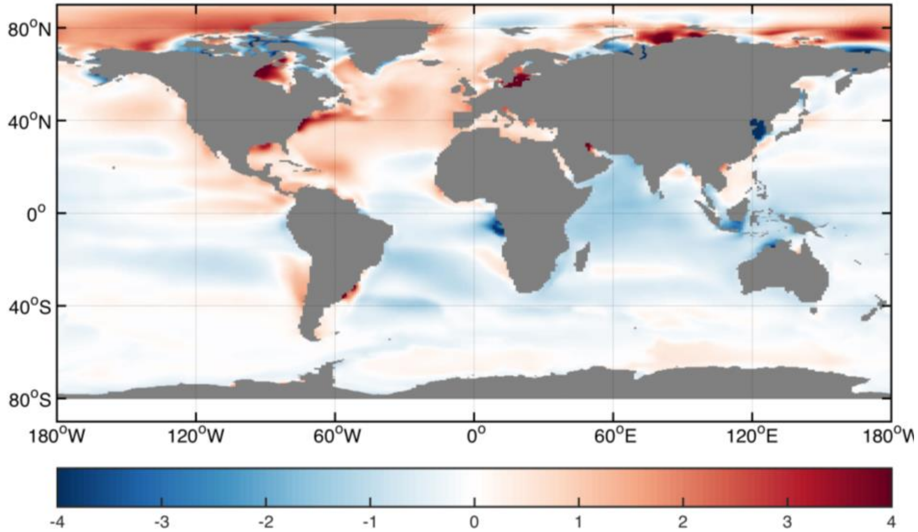


Old GM

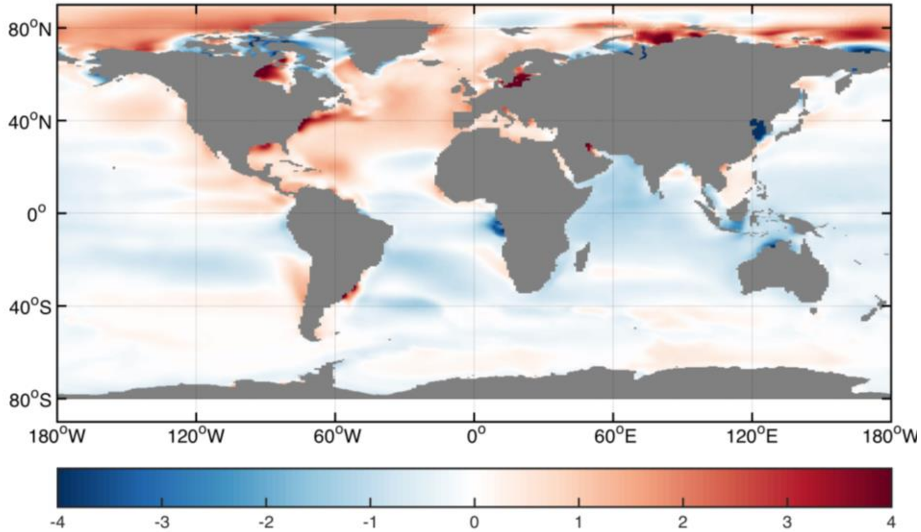


New GM

Coupled: Model SSS – WOA09

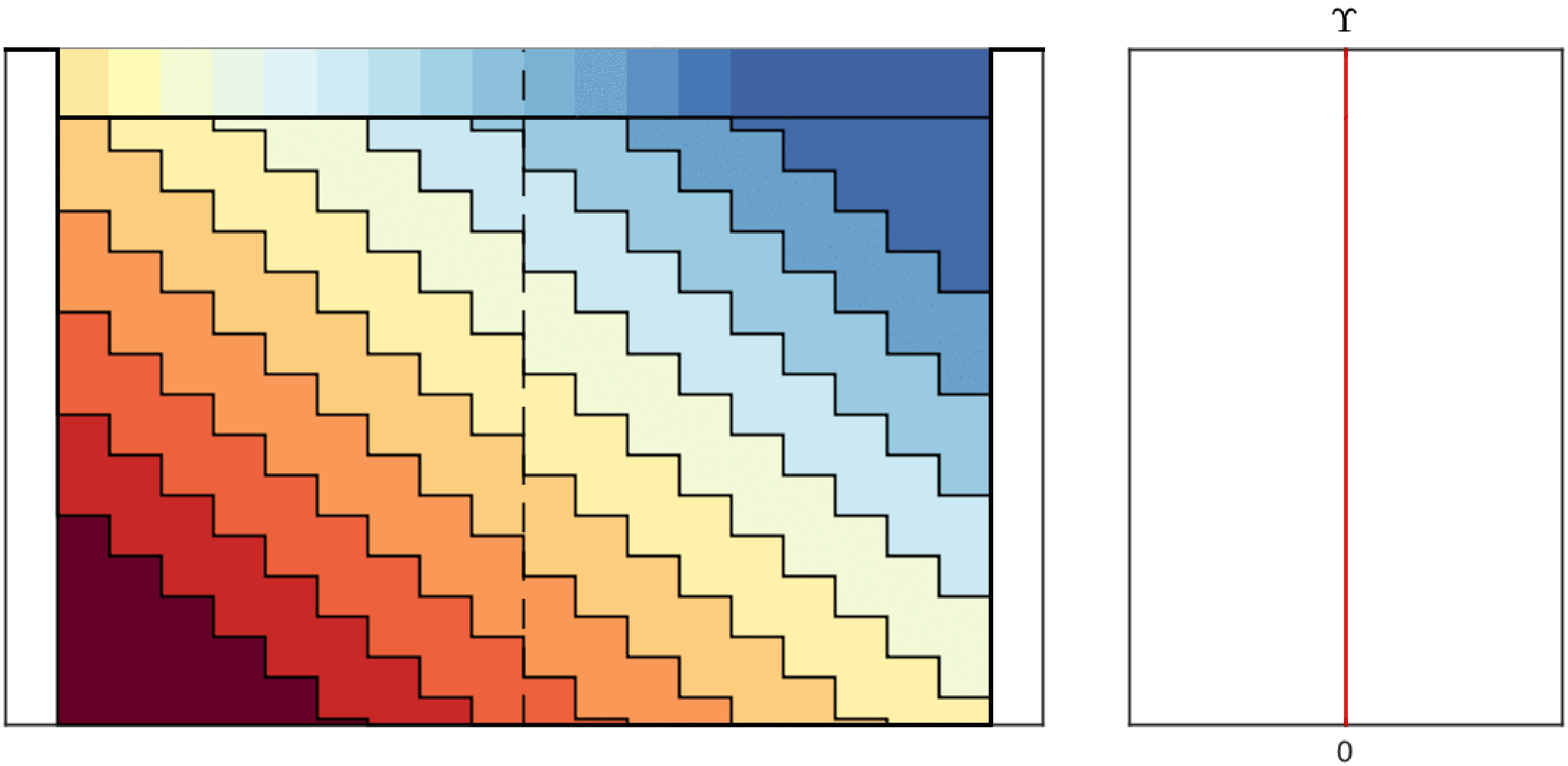


Old GM



New GM

Eddy-induced transport following the approach by Ferrari et al. (2010). A variable density bulk ML is present. Hydrostatic instabilities are removed and the ML depth is restored.



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