A comparison of sequential data assimilation schemes for ocean prediction with HYCOM

Twin Experiments

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Assimilation Schemes for HYCOM

- Multivariate Optimal Interpolation (MVOI)
  J. A. Cummings; Lorenc 1981, Daley 1991, Cummings 2005
- Ensemble Optimal Interpolation (EnOI)
  F. Counillon; Evensen 2003, Oke 2002
- Ensemble Reduced Order Information Filter (EnROIF)
  T. M. Chin; Chin etal 1999,01
- Fixed basis variant of the SEEK filter (SEEK)
  J. M. Brankart and P. Brasseur; Pham et al. 1998

Objective: To assess the performance of these schemes in identically configured twin experiments assimilating synthetic altimeter and surface temperature obs.

Domain & Model Configuration: 1/12° Gulf of Mexico nested in the 1/12° North Atlantic

Data Update Equation

Common linear formula for updating the model-forecast $\mathbf{x}^f$ to obtain data-analysis $\mathbf{x}^a$:

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}(\mathbf{y} - \mathbf{H}\mathbf{x}^f) \tag{1}$$

$\mathbf{y}$ is the data to be assimilated
$\mathbf{H}$ is the observation operator
$\mathbf{K}$ is an optimization parameter often called the *gain matrix*

$$\mathbf{K} = \mathbf{P}^f\mathbf{H}^T(\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R})^{-1} \tag{2}$$

$\mathbf{P}^f$ is the forecast error covariance
$\mathbf{R}$ is the observation error covariance

Main difference among the four methods is in the

numerical representation of $\mathbf{P}^f$

MVOI

- $\mathbf{P}^f$ is separated into several variances and correlations

$$\mathbf{C}_h = (1 + s_h)exp(-s_h) \quad (3)$$
$$\mathbf{C}_v = (1 + s_v)exp(-s_v) \quad (4)$$
$$\mathbf{C}_f = (1 + s_f)exp(-s_f) \quad (5)$$
$$\mathbf{C}_b = \mathbf{C}_f\mathbf{C}_h\mathbf{C}_v \quad (6)$$

where $\mathbf{C}_h$ and $\mathbf{C}_v$ are the horizontal and vertical correlations and s_h and s_v are scaled distances.

- analysis on z levels (42 zlevels between 0-2500 m)
- state variables: U, V, T, S and intf. Pressure
- dynamical method (Cooper-Haines 1996) for vertical projection
- simultaneous 3D analysis
- obs errors uncorrelated

EnOI

In EnOI, the forecast covariance matrix is essentially the *sample covariance* of an ensemble of model states

$$\mathbf{P}^f_{\text{EnOI}} = \frac{1}{M-1}\sum_{m=1}^{M}\left(\mathbf{x}^f_m - \overline{\mathbf{x}}^f\right)\left(\mathbf{x}^f_m - \overline{\mathbf{x}}^f\right)^T \qquad (7)$$

where $\mathbf{x}^f_m$ is the m^{th} sample of the forecast ensemble, $\overline{\mathbf{x}}^f$ is the ensemble mean, and M is the number of samples. The analysis update is computed as:

$$\mathbf{x}^a = \mathbf{x}^f + \alpha\mathbf{P}^f\mathbf{H}^T(\alpha\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R})^{-1}(\mathbf{y} - \mathbf{H}\mathbf{x}^f) \qquad (8)$$

where $\alpha \in (0,1]$ is a parameter introduced to allow different weights for the ensemble and measurements.

State Varaibles: UT, VT, UB, VB, PB, DP, T, S, SSH, SST

Vertical Projection: static correlations; analysis in observation space

EnROIF

EnROIF uses a *Markov random field* (MRF) to model the forecast error process as

$$e(i,j) \;=\; \sum_{(\triangle i, \triangle j)\,\in\,\mathcal{N}} \gamma(i,j,\triangle i,\triangle j)\, e(i-\triangle i, j-\triangle j) \;+\; \delta(i,j) \qquad (9)$$

where $\mathcal{N}$ specifies a set of local grid locations, γ is the regression coefficient (small matrix), and $\delta(i,j)$ is a white noise with unit variance. Computation for the state analysis $\mathbf{x}^a$ in ROIF is performed as

$$\mathbf{L}^a(\mathbf{x}^a - \mathbf{x}^f) \;=\; \mathbf{H}^T\mathbf{R}^{-1}(\mathbf{y} - \mathbf{H}\mathbf{x}^f) \qquad (10)$$

where the sparsely banded analysis information matrix $\mathbf{L}^a = \mathbf{L} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}$ is numerically inverted.

State Varaibles: DP, SSH, SST

Vertical Projection: static correlations; 2D vertically decoupled analysis

Fixed basis SEEK

In SEEK filter, $\mathbf{P}^f$, is assumed to be of low rank, M and is represented by dominant modes of *empirical orthogonal functions* (EOFs) as

$$\mathbf{P}^f_{\text{SEEK}} = \frac{1}{M-1}\sum_{m=1}^{M}\mathbf{S}^f_m\mathbf{S}^{fT}_m \tag{11}$$

where $\mathbf{S}^f_m$, $m = 1, \ldots, M$, are the M most dominant EOF modes. In

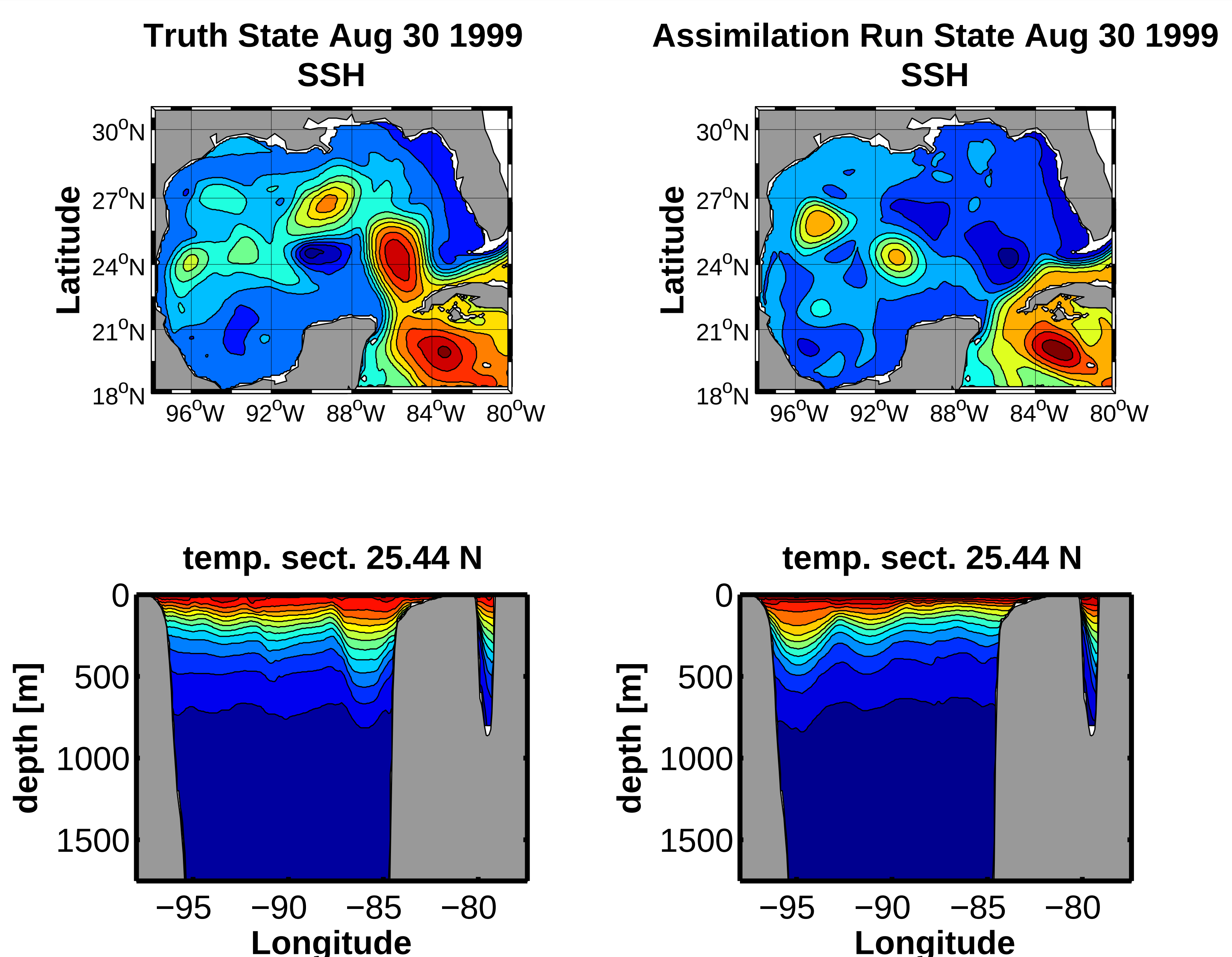
the SEEK analysis the Kalman Gain is rewritten using the Sherman-Morrrison-Woodberry matrix identity as

$$\mathbf{K} = \mathbf{S}^f[\mathbf{I} + \left(\mathbf{H}\mathbf{S}^f\right)\mathbf{R}^{-1}\left(\mathbf{H}\mathbf{S}^f\right)^T]^{-1}\left(\mathbf{H}\mathbf{S}^f\right)\mathbf{R}^{-1} \tag{12}$$

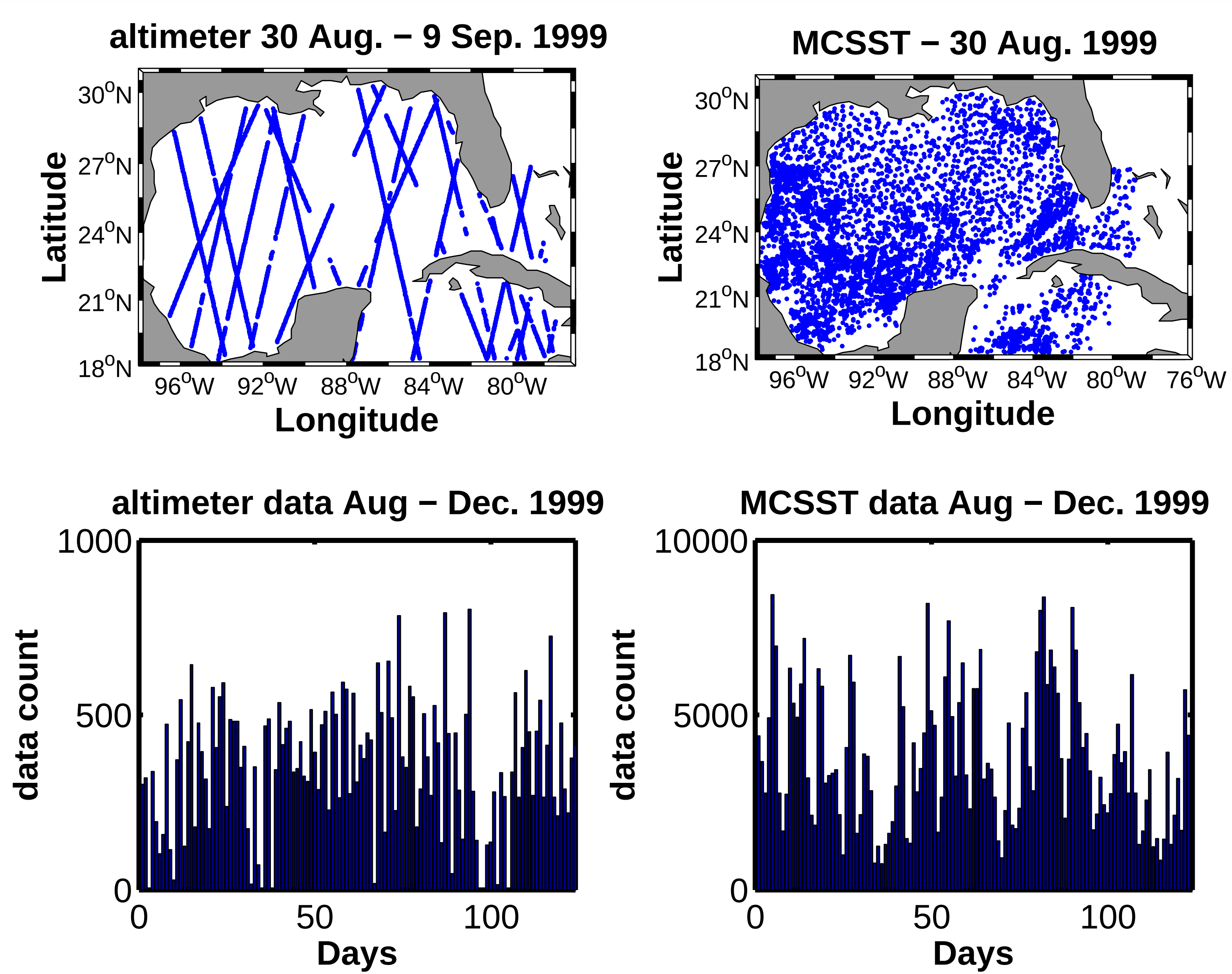
State Varaibles: UT, VT, UB, VB, PB, DP, T, S, SSH, SST

Vertical Projection: static correlations; analysis in reduced space

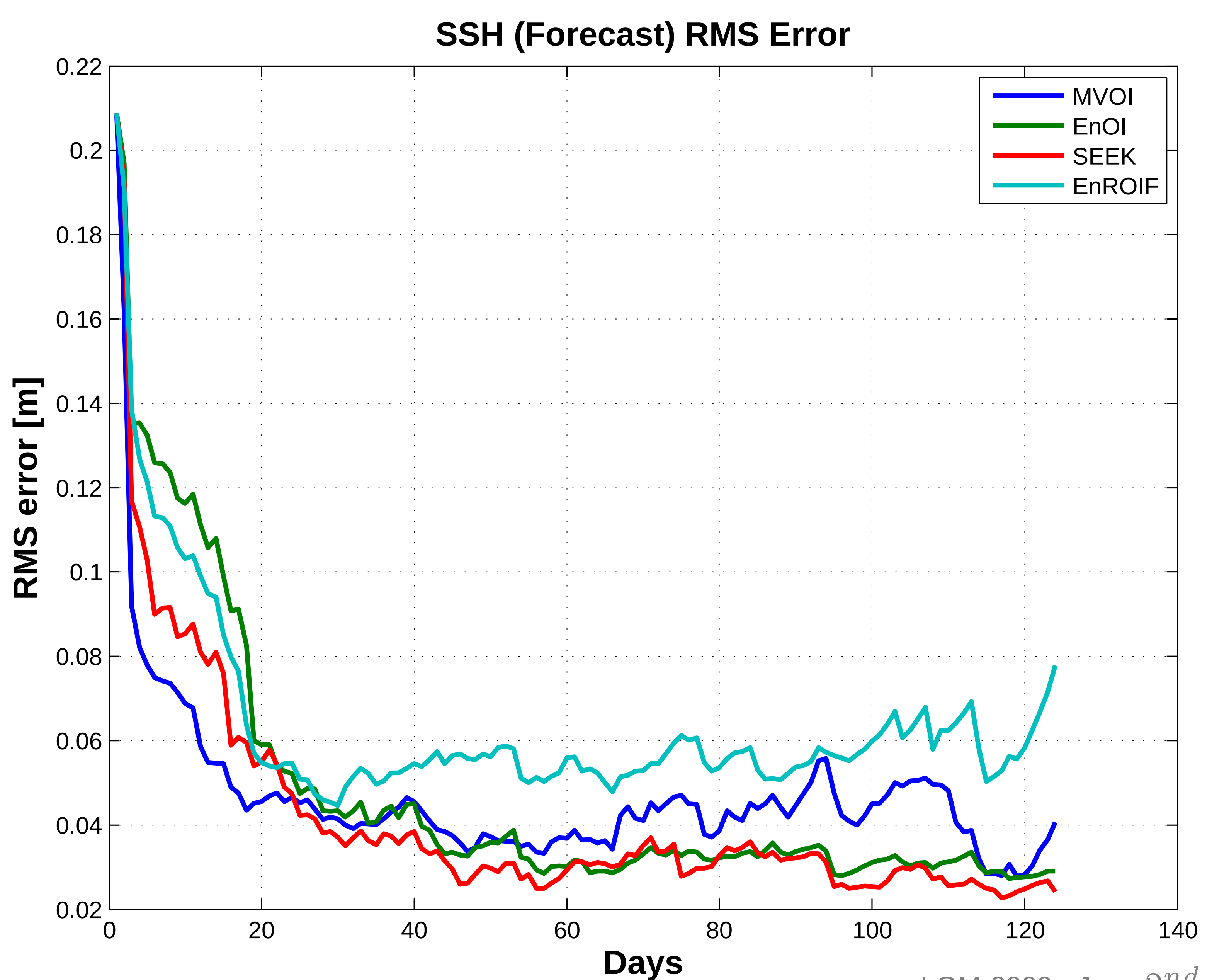
Twin Experiments: Initial States



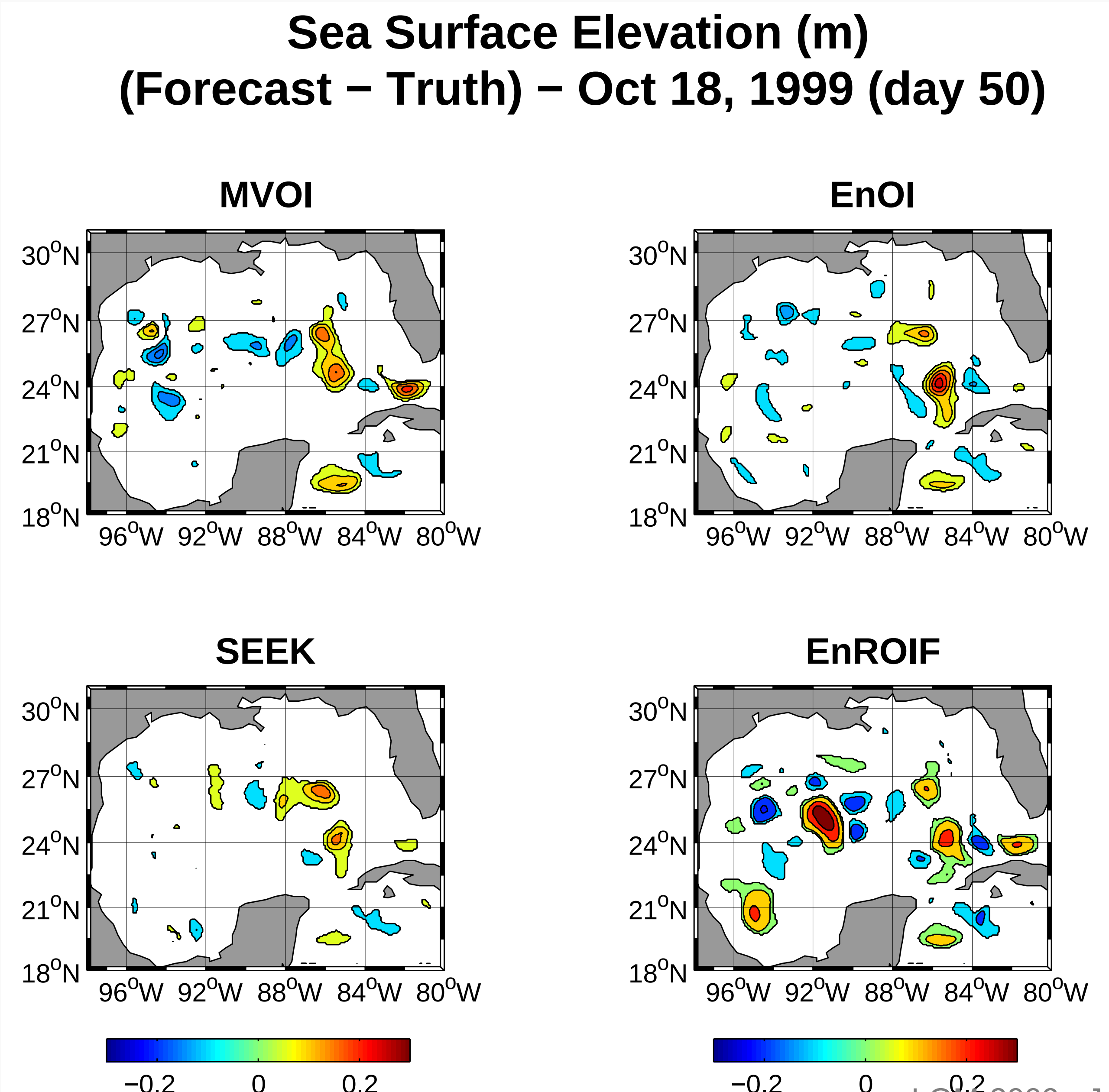
Twin Experiments: Data



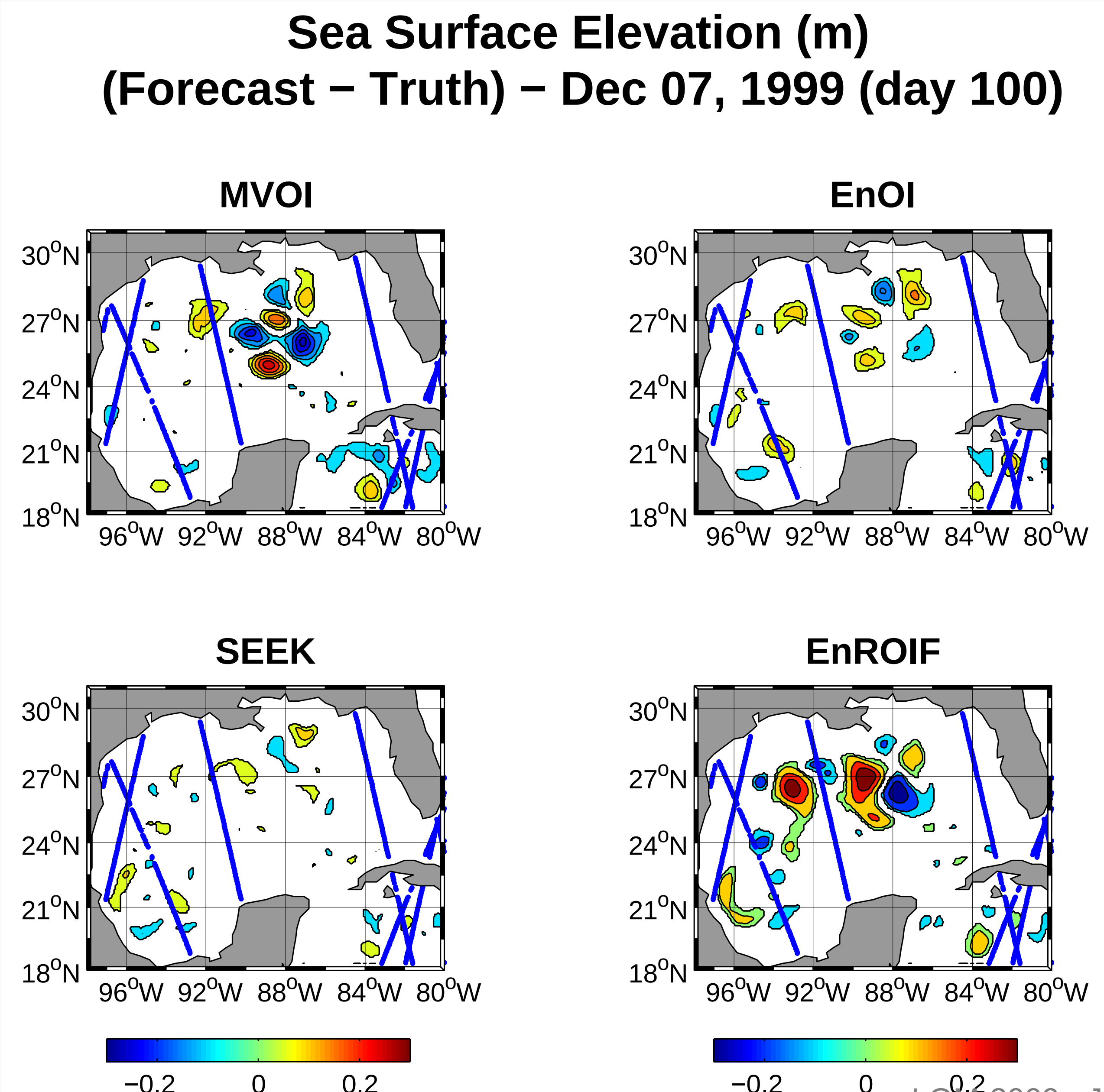
Twin Experiments: SSH



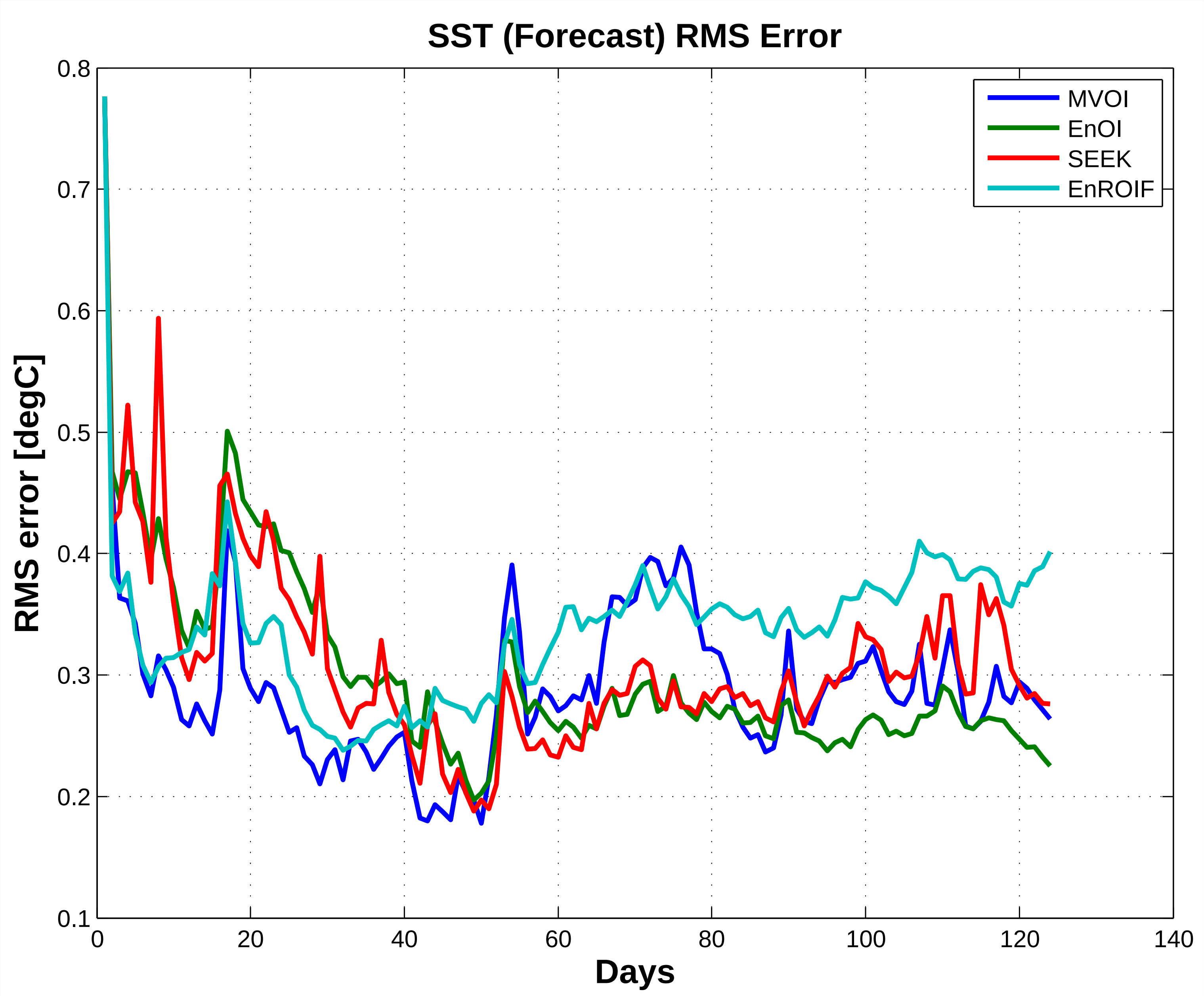
Twin Experiments: SSH



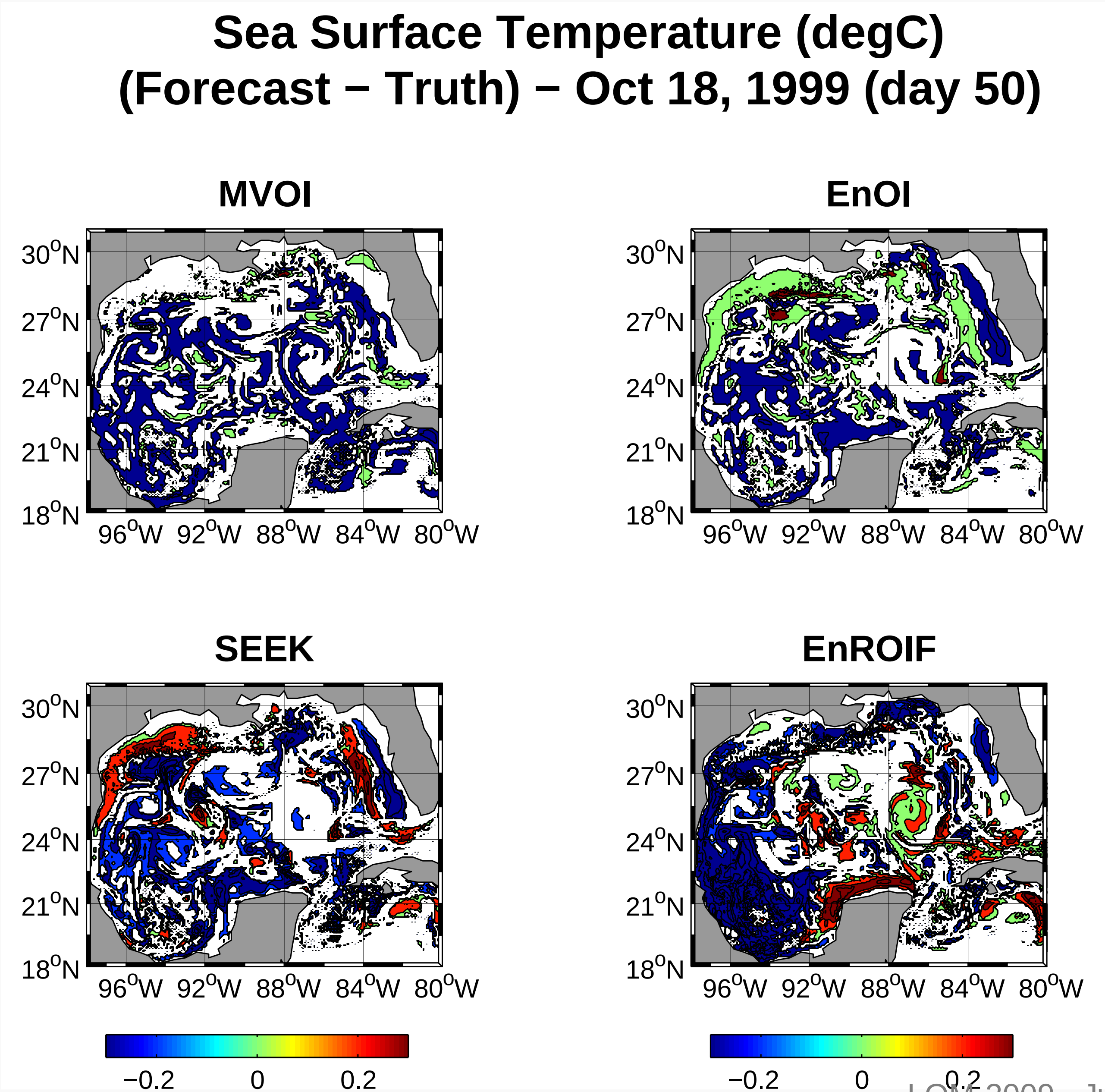
Twin Experiments: SSH



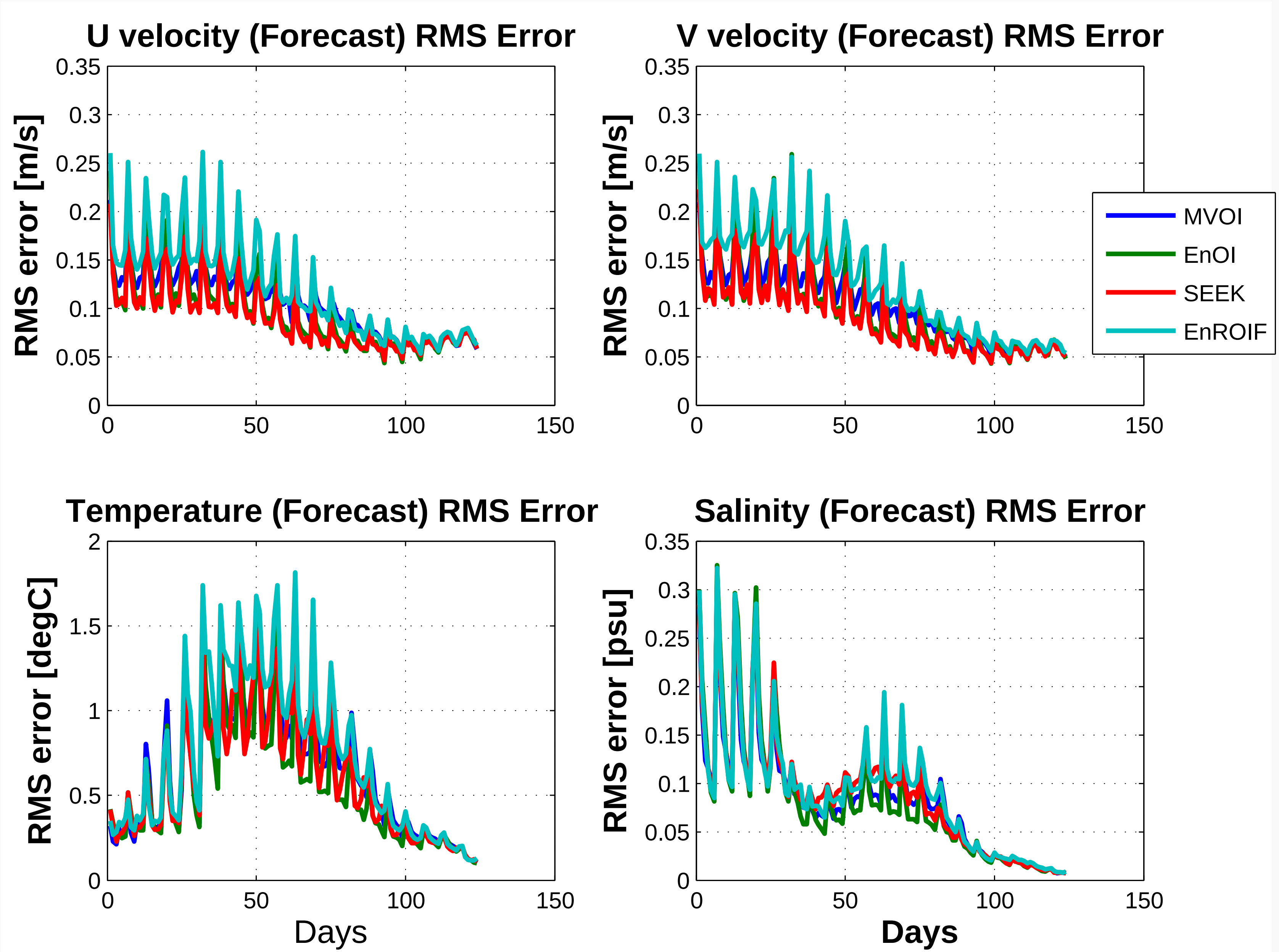
Twin Experiments: SST



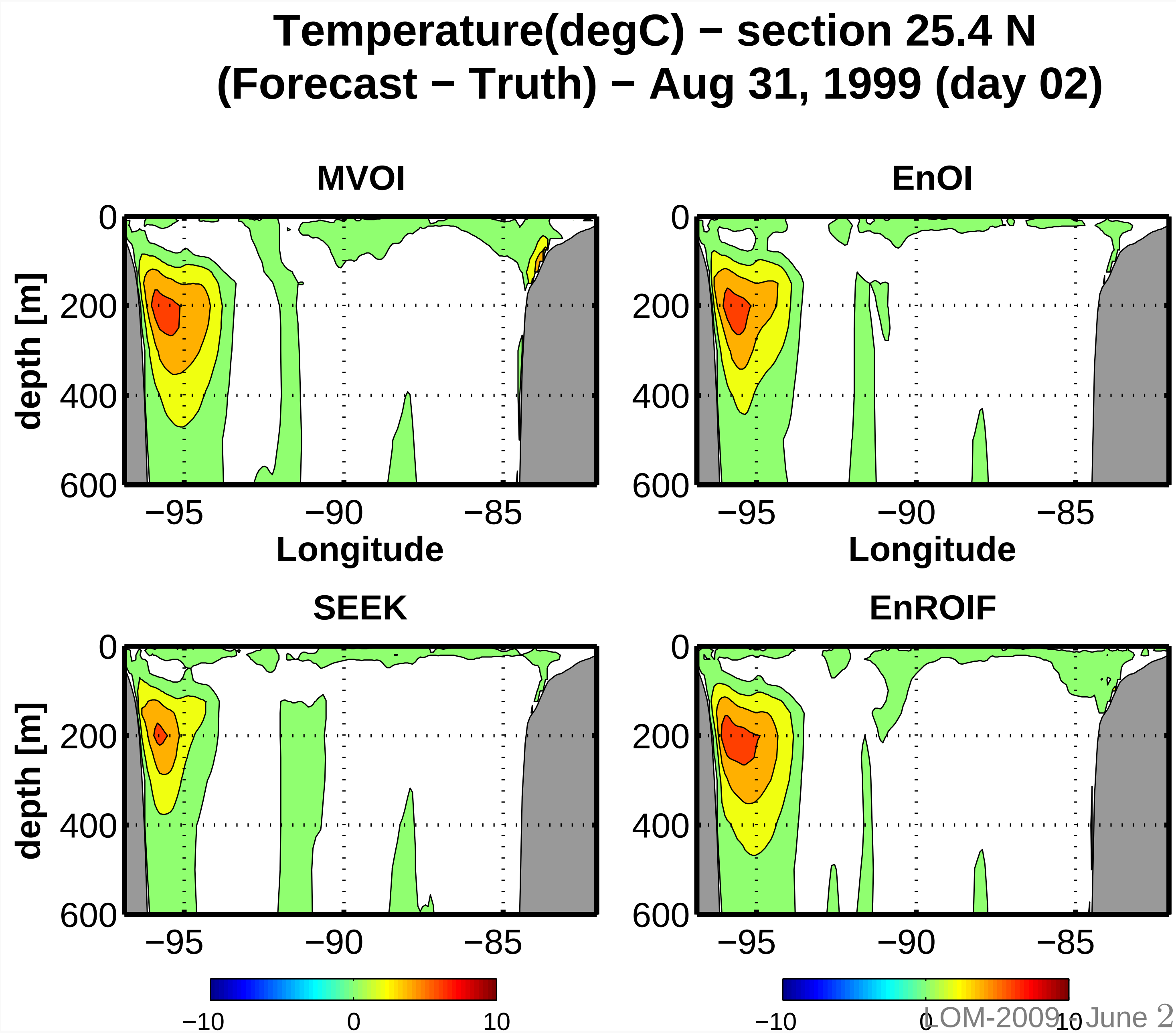
Twin Experiments: SST



Twin Experiments: Unobserved Variables [u,v, t,s]



Twin Experiments:



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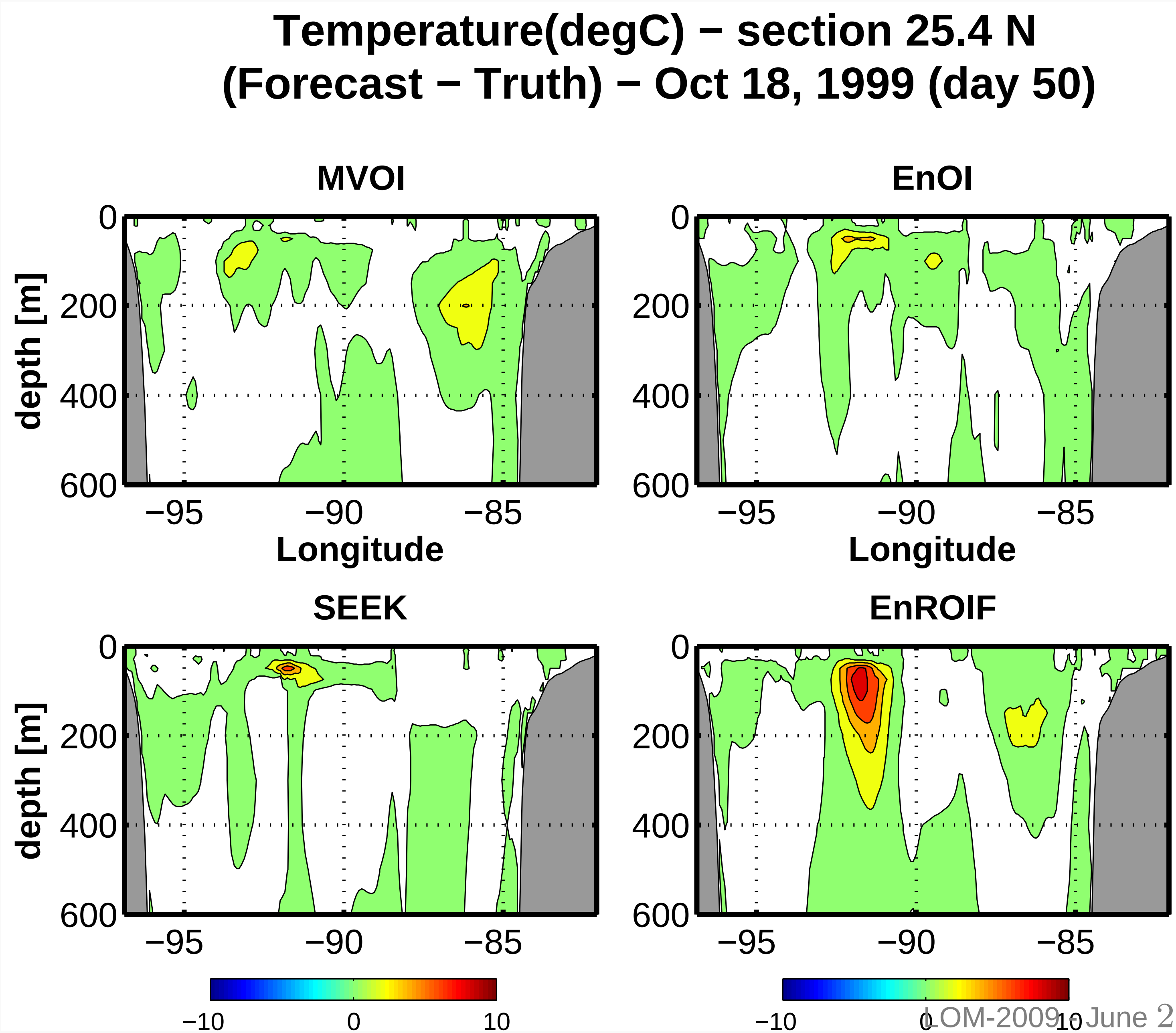

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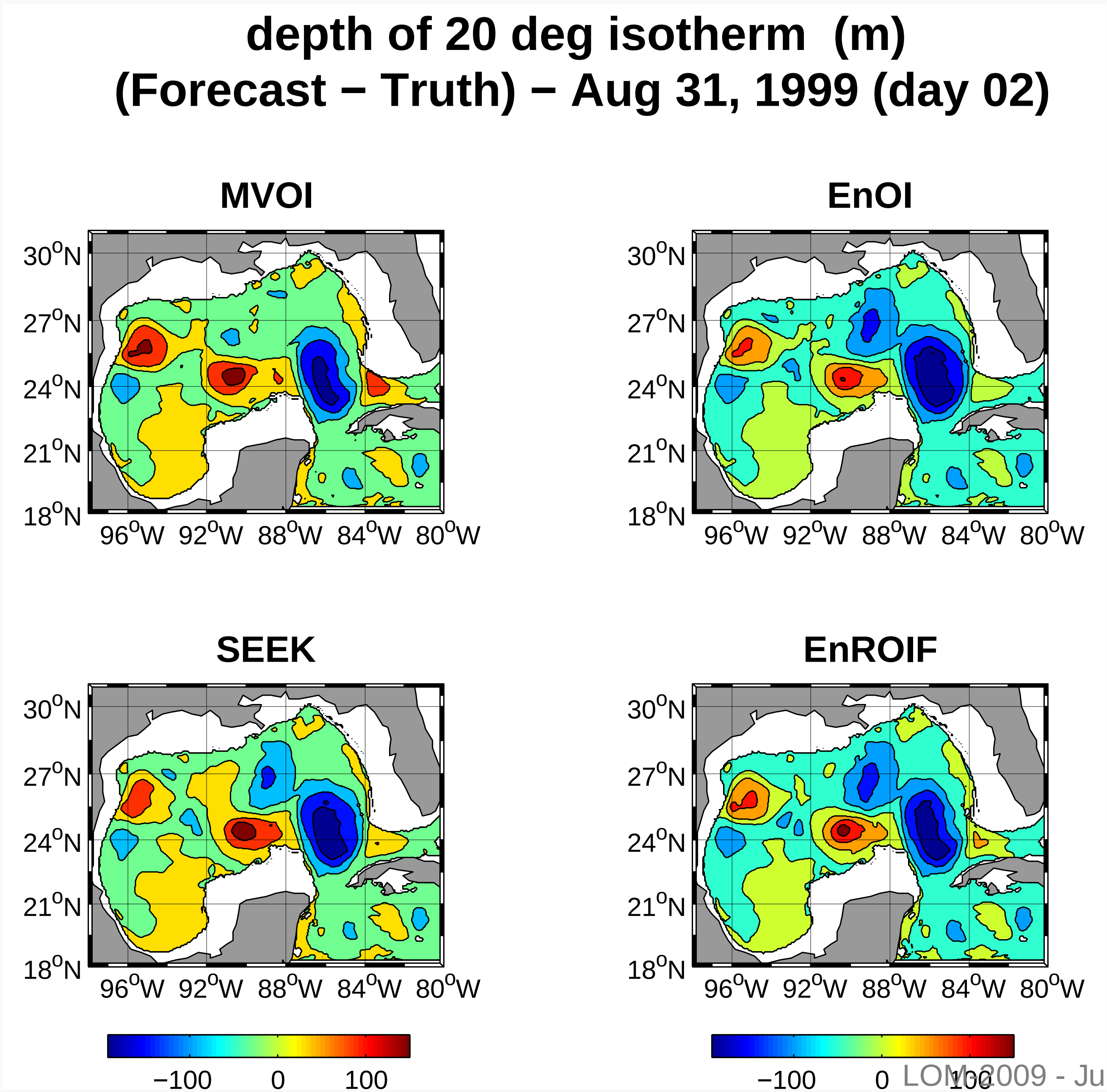

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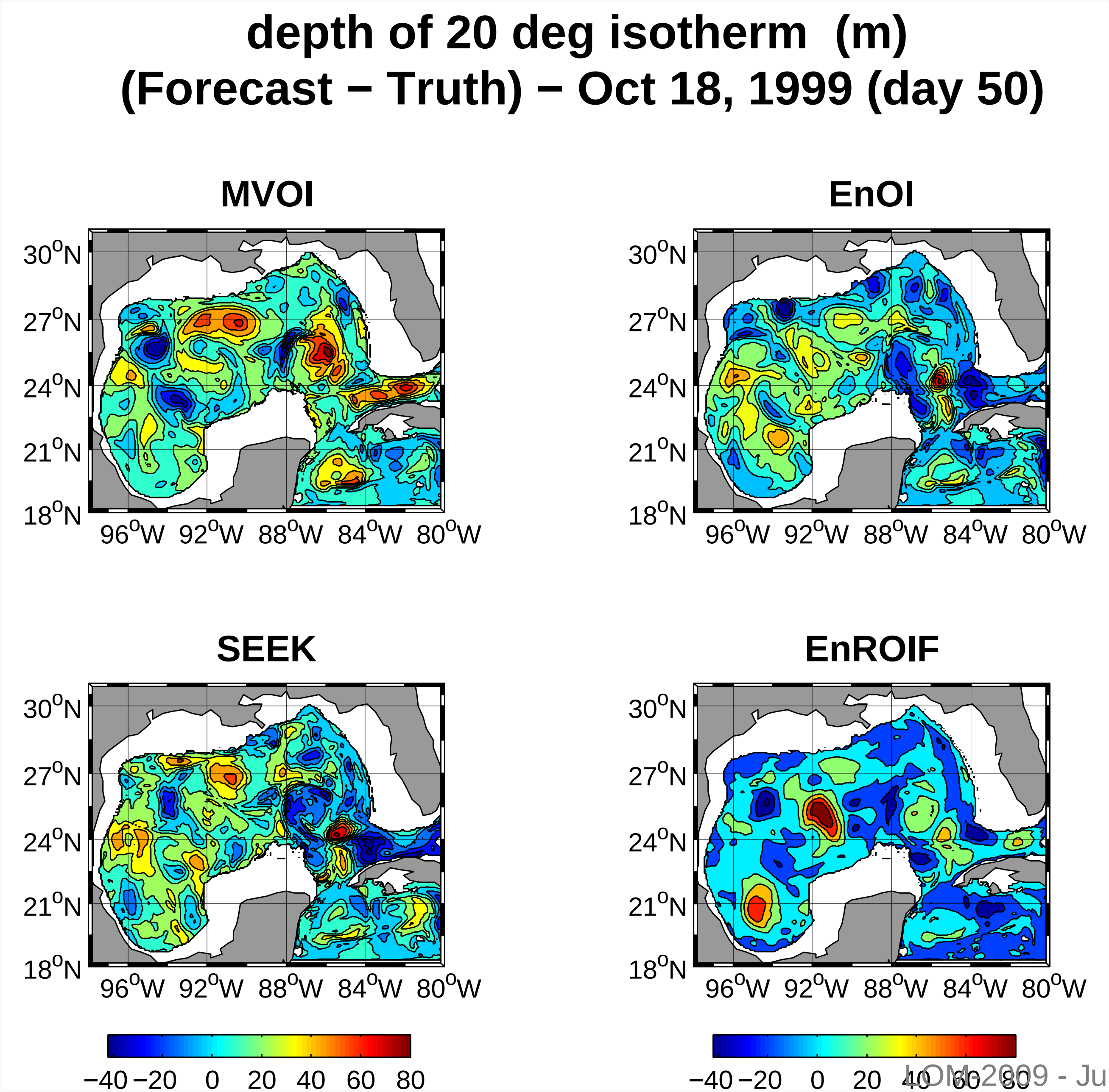
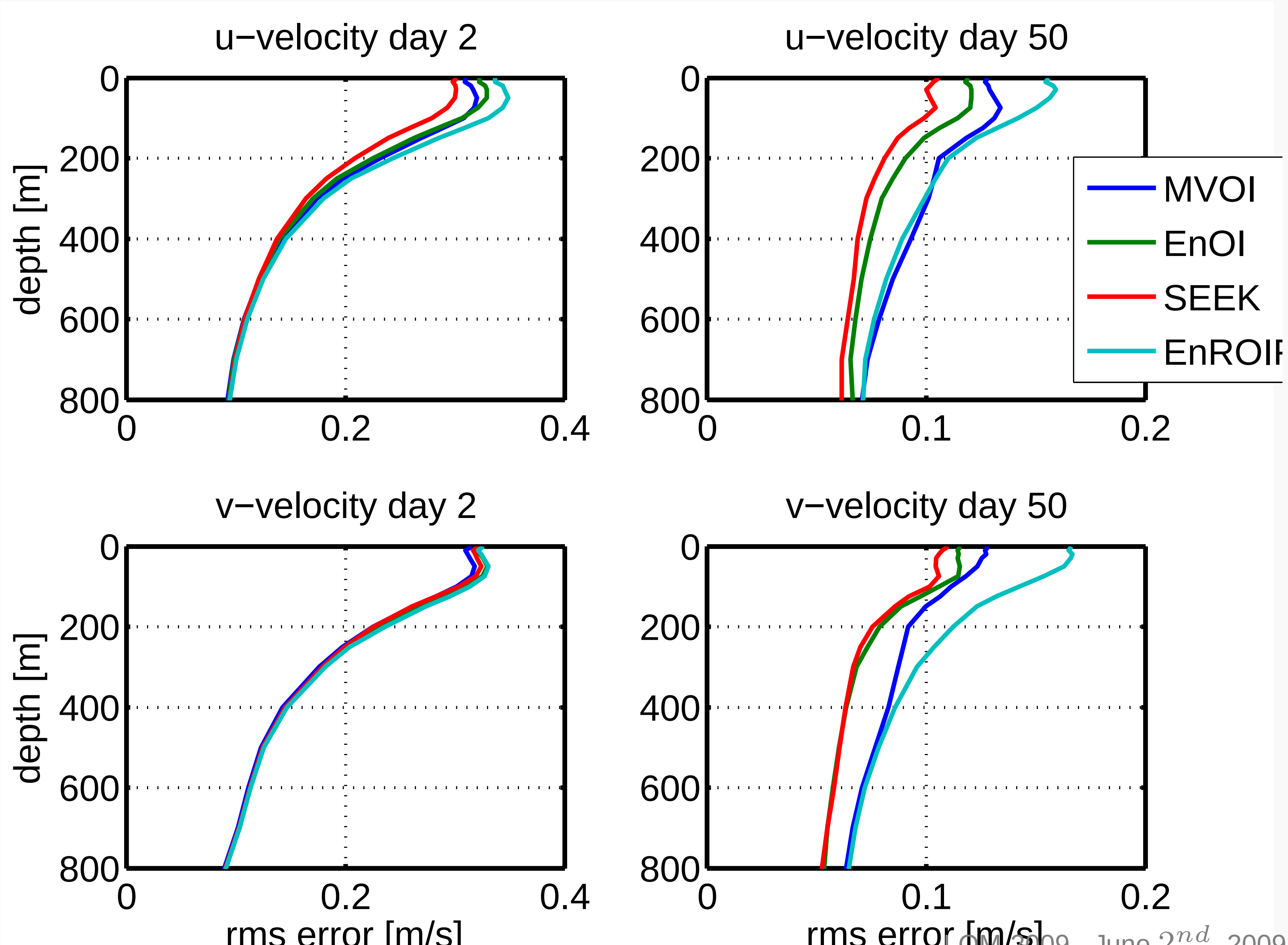
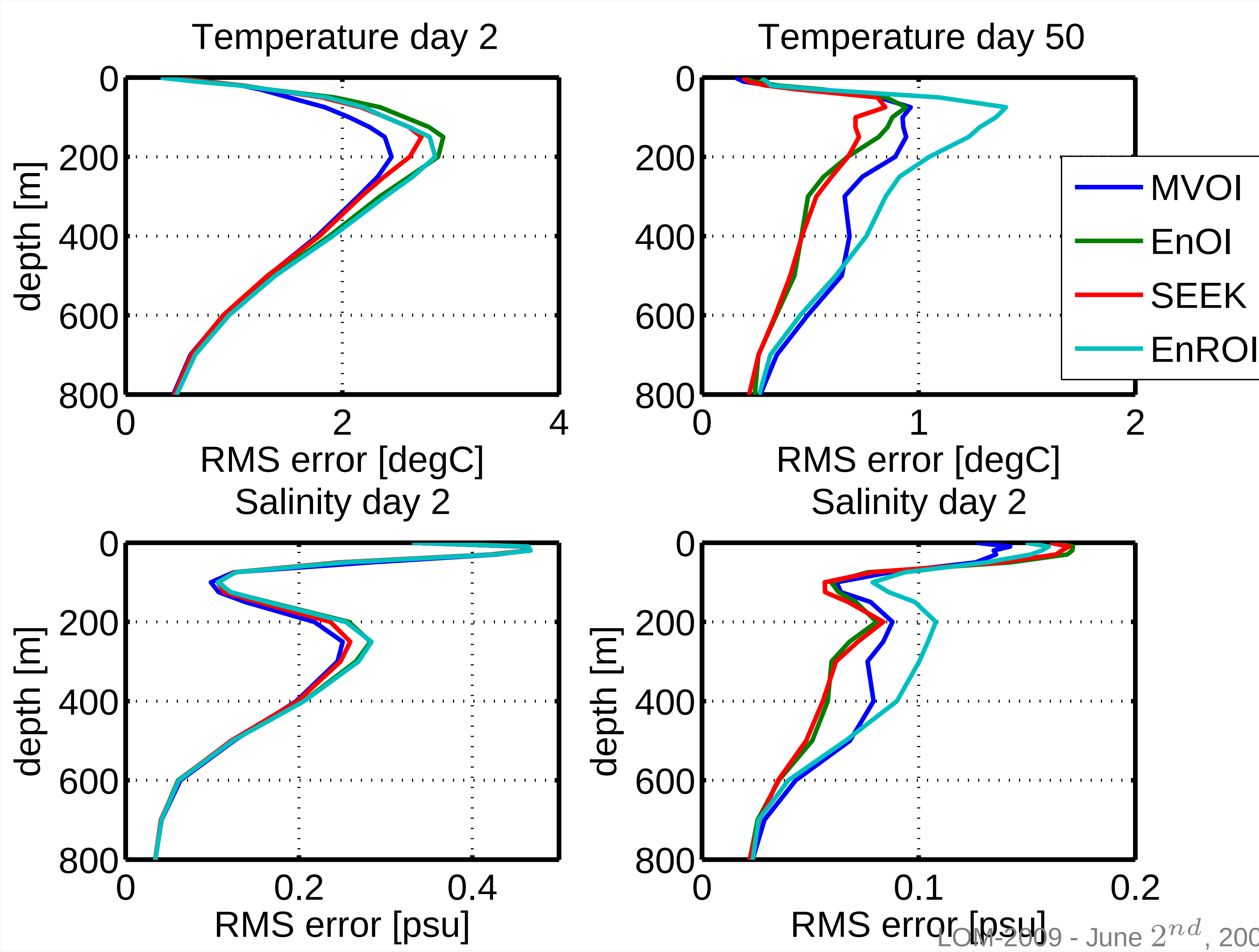


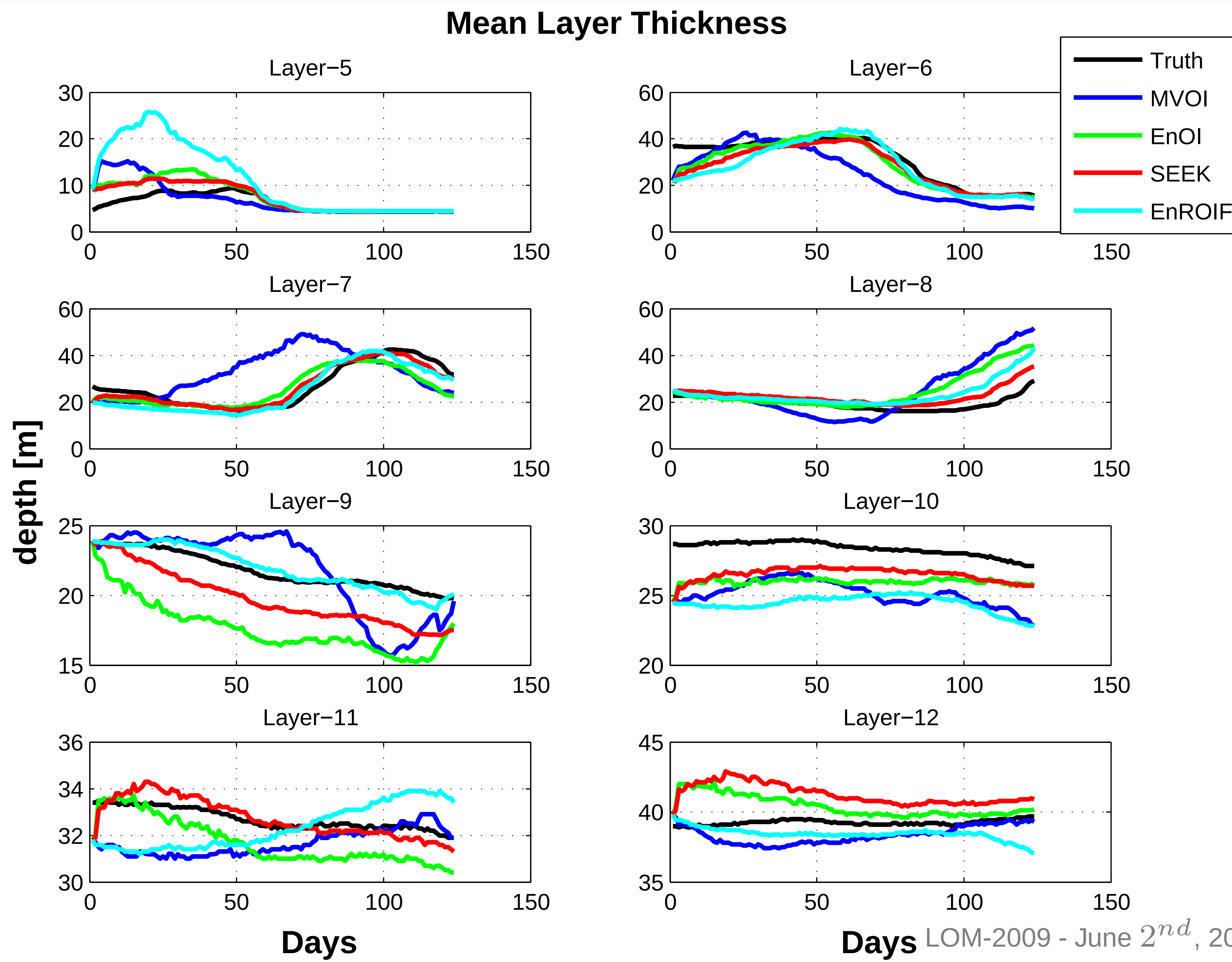
Twin Experiments: 3D U&V



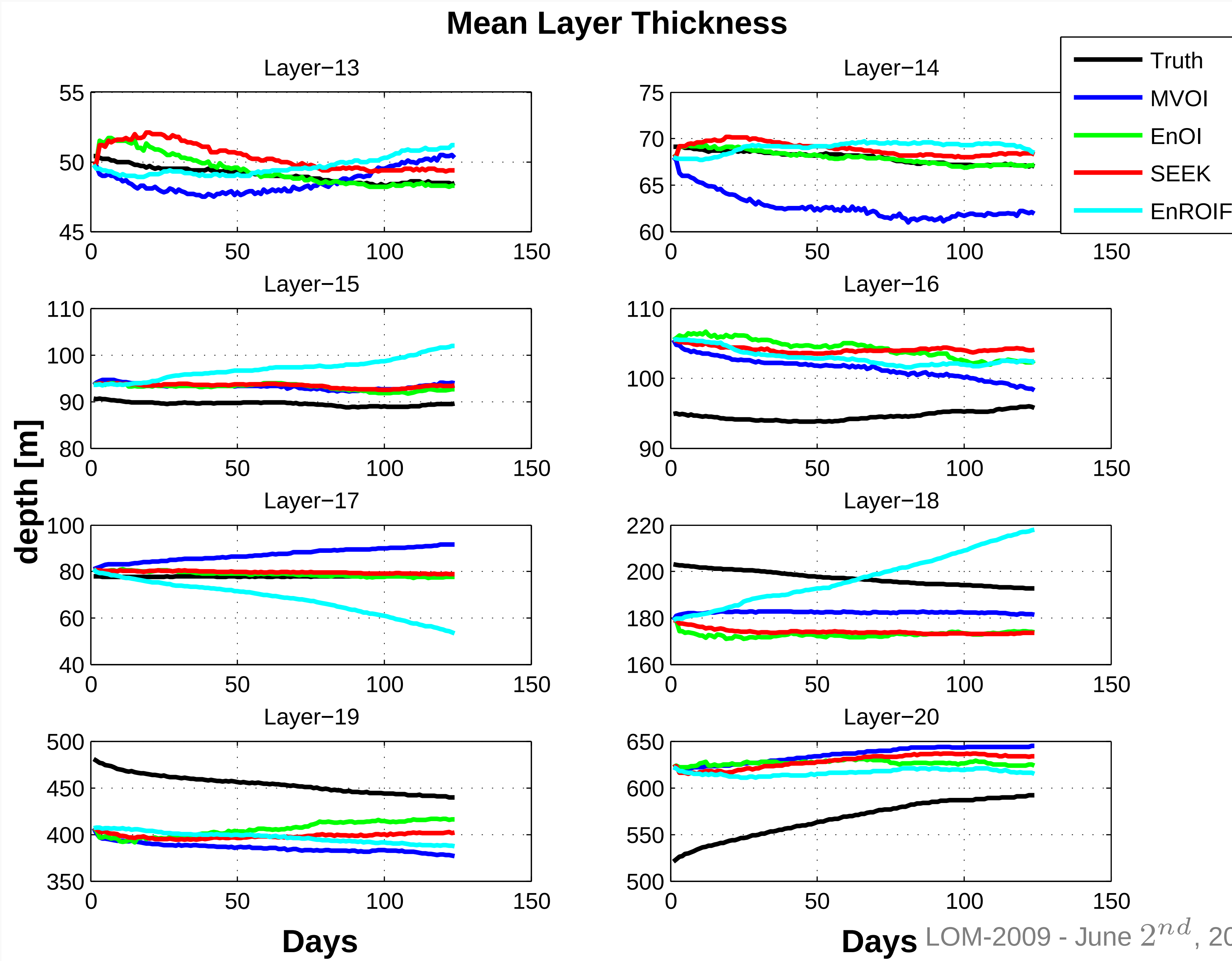
Twin Experiments: 3D T&S



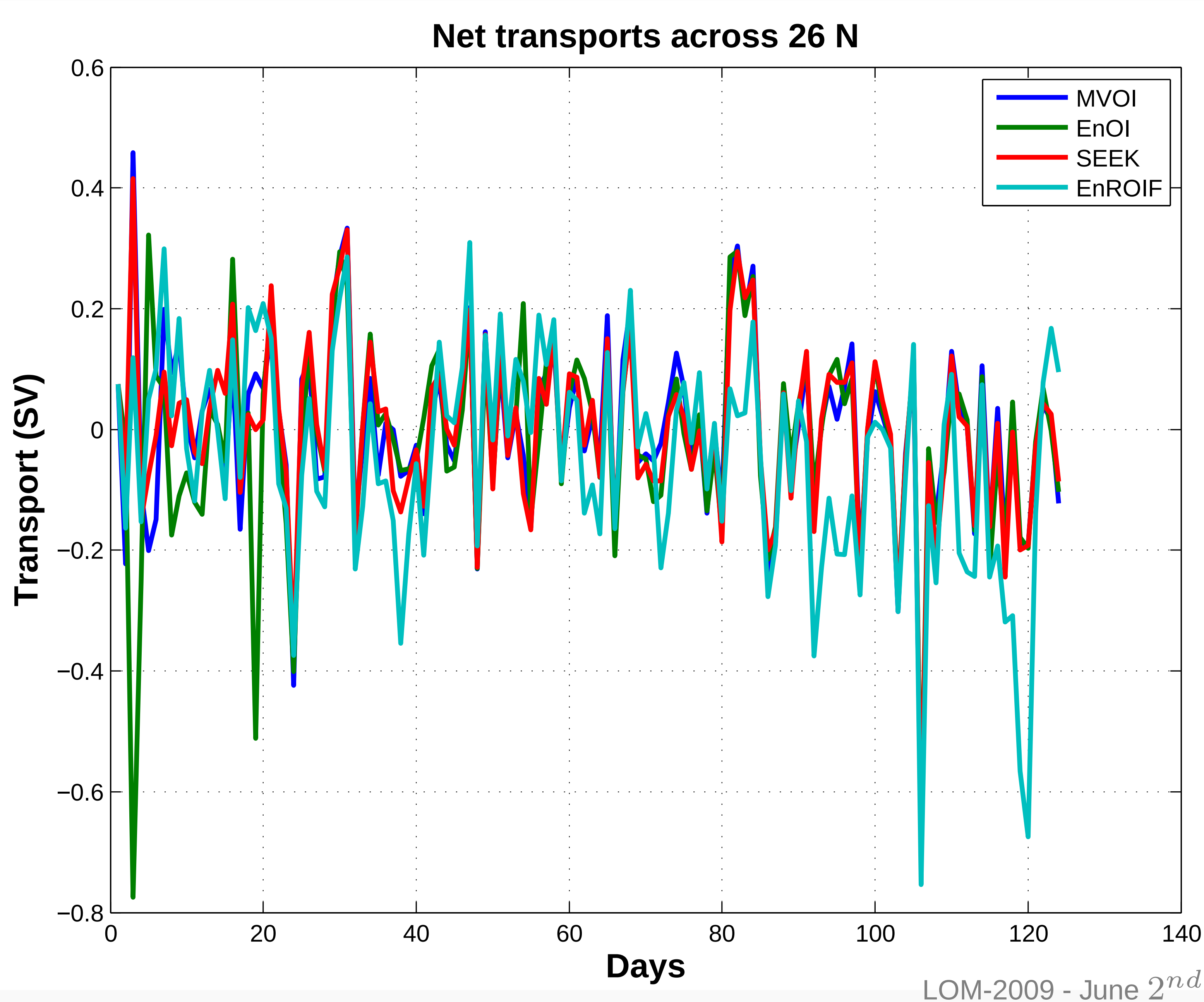
Twin Experiments: Layer Thickness



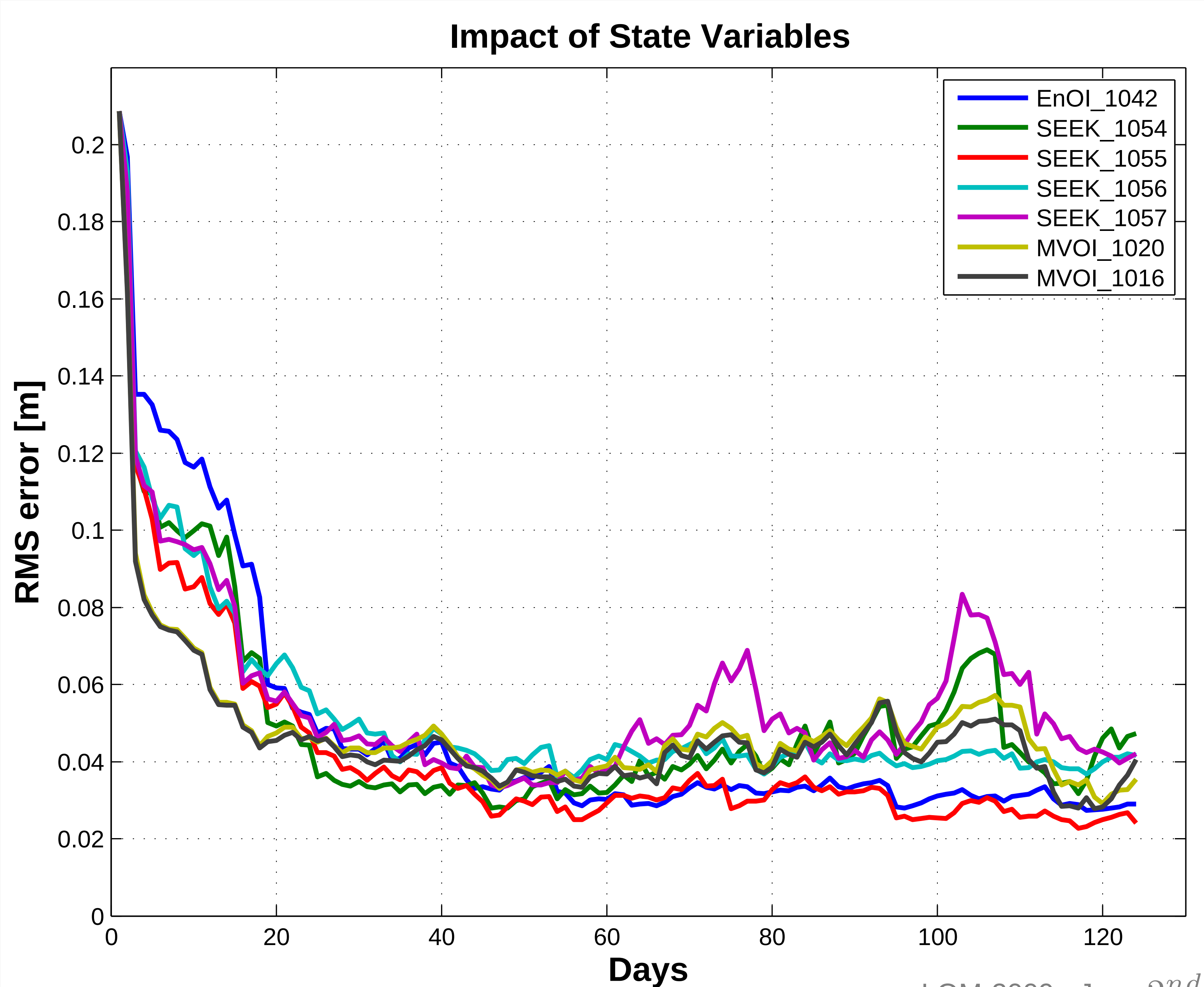
Twin Experiments: Layer Thickness



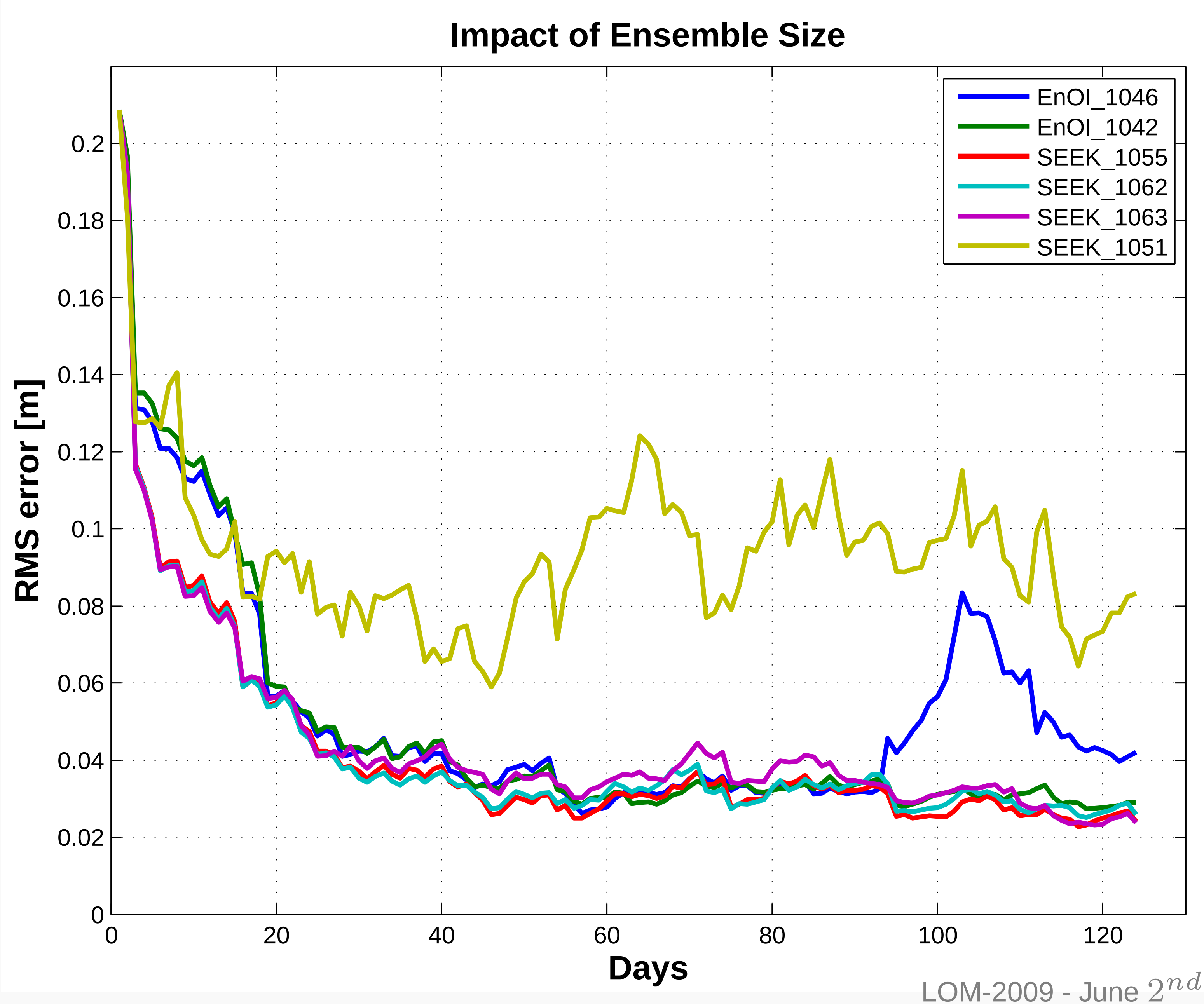
Twin Experiments: Net Transports



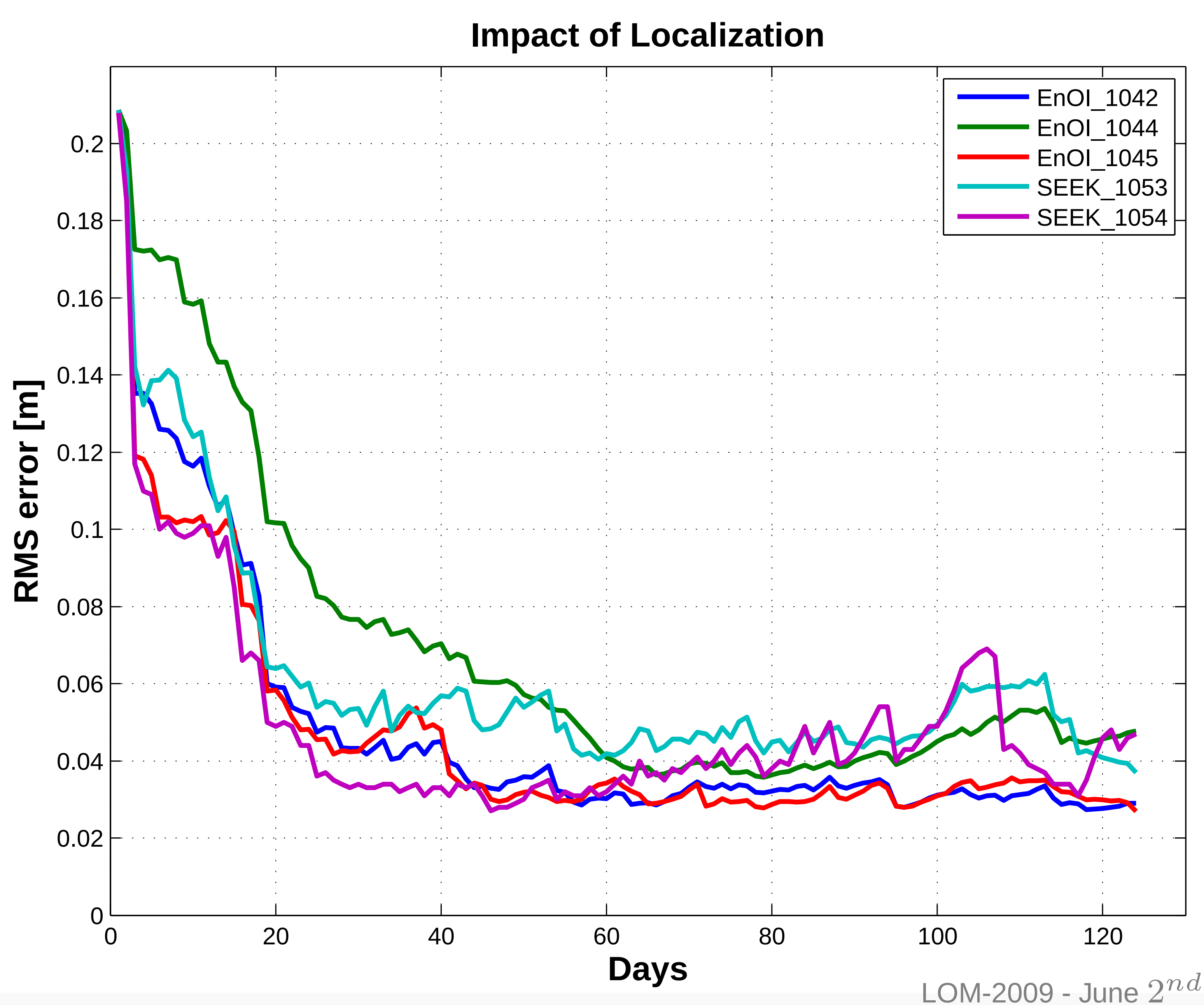
Twin Experiments: State Variables



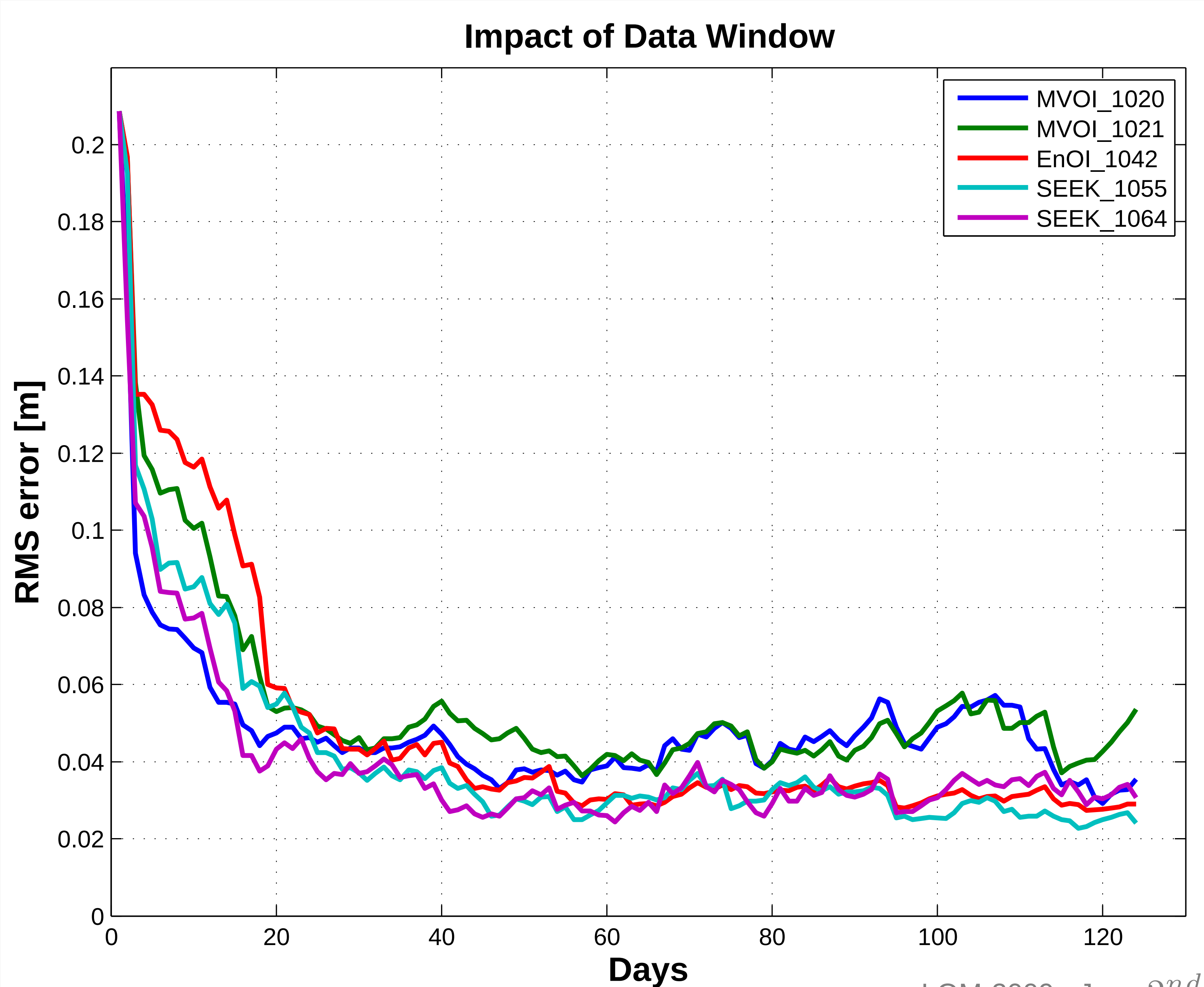
Twin Experiments: Ensemble Size and Covariance Rank



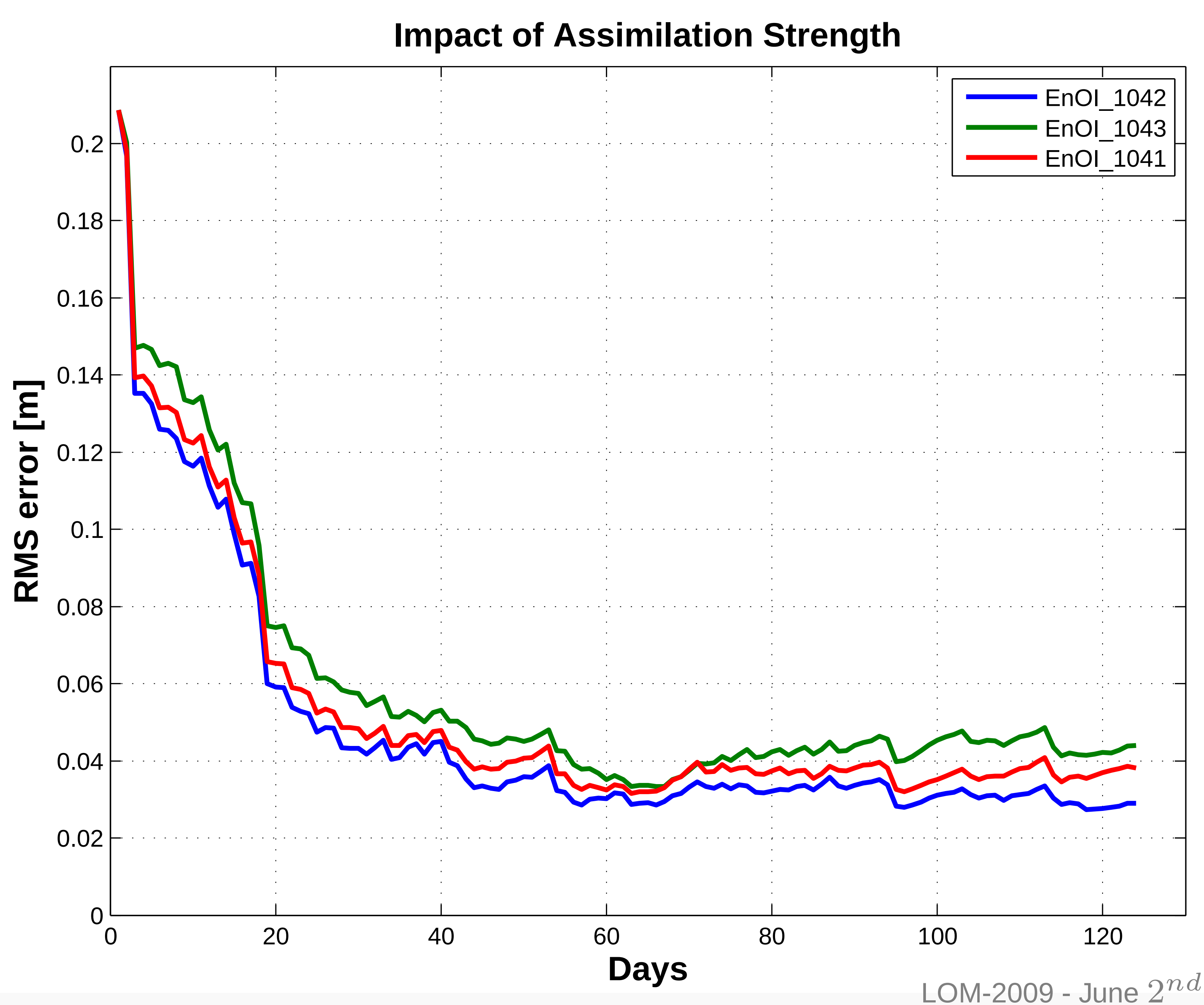
Twin Experiments: Localization



Twin Experiments: Data Window



Twin Experiments: Assimilation Strength



Computational Costs

Scheme	Scalability	Memory Req.	Typical Wall Clock/update
MVOI	$O(m^3/V)$	–	22 min
EnOI	$O(m^3)$	nN	31 min
EnROIF	–	sN	8 min
SEEK	$O(r^3)$	nN	14 min

n ==> no of members in the ensemble
N ==> size of the forecast model
s ==> size of the MRF neighborhood
r ==> covariance rank (== no of members in our experiments)
v == no of analysis volumes in MVOI
all experiments were done on 8 core/2.6 GHz Intel/12GB RAM)

Summary

- All schemes are able to reduce errors in both observed and unobserved variables.
- 3D multivariate (MVOI, EnOI, SEEK) are clearly better than decoupled 2D scheme used currently in EnROIF
- Best performance is obtained when all state variables are used in the estimation space
- Subsurface correction based on correlations(EnOI and SEEK) are better balanced and seem more effective than the dynamical method(MVOI)
- The nearly identical performance of EnOI and SEEK is both expected and reassuring
- Several parameters (ensemble size, covariance rank, radius of localization etc.) have to be explored for a given problem to find a good solution that is also cost effective.