Interannual variability of upper ocean vorticity balances in the Gulf of Alaska

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Abstract. A high-resolution numerical ocean model is used to examine the interannual variability of the upper ocean vorticity budget in the Gulf of Alaska. A circulation equation is derived for a layer representative of the upper ocean. The equation is a balance of the area integral of vorticity over the domain with the time- and area-integrated vorticity flux through the layer top, bottom, and lateral boundaries and dissipation. Time series of each component of the equation are constructed and examined for interannual variability. Other native and derived model variables are analyzed to facilitate explanation of observed interannual variability. The model data show that interannual fluctuations in the time rate of change of the circulation in the Gulf of Alaska are small compared to the rate at which the dominant sources and sinks feed and drain vorticity from the domain. Variability on interannual timescales is predominantly due to atmospheric and oceanic teleconnections originating with the El Niño–Southern Oscillation. Direct forcing of the wind on the ocean is the strongest driving mechanism for interannual variability of the gyre circulation. However, large fluctuations in the wind forcing are partially balanced by other processes. El Niño events excite downwelling coastal Kelvin waves that propagate northward along the eastern continental margin of the northeast Pacific Ocean. Arrival of the Kelvin waves at the peak of the annual cycle of the Alaskan Current triggers baroclinic instabilities that result in the formation of a coast-wide train of large anticyclonic eddies. The more southerly of these eddies tend to propagate to the southwest, advecting negative vorticity out of the Gulf of Alaska. This net gain of vorticity partially balances the contemporaneous reduction of vorticity input to the region by the wind stress curl. Excess vorticity is advected out of the region in the Alaskan Stream to maintain the circulation over longer timescales.

1. Introduction

The Alaskan Gyre is the most prominent oceanographic feature in the Gulf of Alaska (Plate 1). It is approximately bounded on the south by the eastward flowing Pacific Current (~45°N), which separates at the continental margin into the southward flowing California Current and the poleward flowing Alaska Current. The latter current ambles northwestward until it curves around the southern coast of Alaska and accelerates west of Kodiak Island, becoming the Alaskan Stream. The timing and position of recirculation filaments connecting the Alaskan Stream to the Pacific Current vary according to local wind stress curl [Thomson, 1972].

The cyclonic circulation of the gyre is wind-driven. The peak of the annual cycle occurs during winter when the cyclonic Aleutian Low dominates the overlying atmospheric circulation. This pattern shifts every summer to one dominated by the North Pacific subtropical high. Emery and Hamilton [1985], Putman [1998], and Smith et al. [1998] have examined interannual variability in the North Pacific atmospheric circulation associated with the El Niño–Southern Oscillation. The variability is manifest as a weakening of the Aleutian Low in the winter prior to an El Niño followed by a rapid strengthening during its mature phase, as well as a change in spatial coverage of the Aleutian Low. Tropically forced coastal Kelvin waves excited during El Niño events propagate northward along the eastern continental margin of the northeast Pacific Ocean and destabilize the Alaskan Current, generating trains of large anticyclonic eddies [Melsom et al., 1999]. The formation and migration of these eddies have recently been observed and well documented [Myak, 1985; Thomson and Gower, 1998; Crawford and Whitney, 1999]. The interannual appearance of these eddies in the Gulf of Alaska is an important aspect of the upper ocean circulation. The present study extends these observations by quantifying the influence of each process playing a role in the upper ocean circulation by examining the balances of terms in the circulation equation in a domain representative of the upper ocean in the Gulf of Alaska. The equation relates the surface integral of depth-averaged vorticity to vorticity flux through pathways into and out of the domain. A discretized version of the equation is solved with gridded model output fields for the region.

The solutions show that interannual variability in vorticity balances is predominantly due to interannual variability in the atmospheric circulation and oceanic teleconnections originating with the El Niño–Southern Oscillation (ENSO). The southward migration of large anticyclonic eddies spawned by El
Niño–induced tropically forced coastal Kelvin waves advect negative vorticity out of the Gulf of Alaska. This net gain of vorticity is inversely correlated with the vorticity input by the wind stress curl. Vorticity advection in the Alaskan Stream drains excess vorticity from the system, resulting in a more slowly fluctuating circulation in the Alaskan Gyre than would be expected by simply examining the variability of the driving mechanism alone.

2. Model

2.1. Model Phenomenology

The Navy Layered Ocean Model (NLOM) is used for this project in order to extend and complement preceding lower-resolution modeling studies of the Gulf of Alaska [Melsom et al., 1999]. The NLOM is the primary isopycnal ocean model used for simulation of open ocean circulation at the Naval Research Laboratory, Stennis Space Center, Mississippi (NRLSSC). The latest version, described by Wallcraft [1991], is a descendant of the primitive equation model developed by Hurlburt and Thompson [1980]. Although both hydrodynamic and thermodynamic versions of the model are available, the most temporally useful data set available at the time of this research was hydrodynamic. The vertically integrated nonlinear model equations of motion for an n-layer hydrodynamic finite depth model are, for $k = 1 \ldots n$:

$$\frac{\partial \mathbf{v}_k}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v}_k + \mathbf{f} \times \mathbf{v}_k + \hat{k} \times f \mathbf{v}_k = -h_k \sum_{l=1}^{n} G_{kl} \nabla(h_l - h_k)$$

$$+ \frac{\tau_{k-1} - \tau_k}{\rho_0} + A_k h_k \nabla v_k + \max(0, w_k) h_{k+1}$$

$$+ [\max(0, -w_k) + \max(0, w_{k-1})] v_k$$

$$+ \max(0, -w_k) v_{k-1} + \max(0, -C_n w_k) (v_k - v_{k-1})$$

$$+ \max(0, C_n w_k) (v_{k+1} - v_k)$$

$$\frac{\partial h_k}{\partial t} \cdot \mathbf{v}_k = \omega_k - \omega_{k-1}.$$

(See notation for descriptions of variables and parameters.)

The finite differences of these equations are computed on a C grid [Messinger and Arakawa, 1976]. The grid spacing of variable fields is constant in latitude and longitude, resulting in higher zonal resolution at higher latitudes. A semi-implicit time scheme is used in the finite depth formulation of the model. Both versions have a free surface and no-slip and kinematic boundary conditions. The finite depth version includes the barotropic mode. See Wallcraft [1991] and Hurlburt and Thompson [1980] for a more complete discussion of model structure and implementation.

NRLSSC has made available model results from NLOM experiment P134-1.7. It is a 15 year six-layer finite depth simulation integrated from January 1981 to December 1995. The data consist of velocity and layer thickness values for each layer at 3.05 day intervals and the wind stress fields with which the model was forced. Although the model is integrated on a C grid, velocity data is interpolated to the $h$ points before output for convenience. Wind stress fields are still colocated with $u$ and $v$ points.

The model domain contains the central and northern Pacific Ocean extending zonally from 109.125°E to 77.203135°W and meridionally from 20.0°S to 62.0°N. No port is included for the Indonesian Throughflow. The choice of domain allows equatorially generated coastal Kelvin waves to influence the northeast Pacific Ocean. Horizontal resolution is 1/16° latitude by 45/512° longitude between like variables, which translates to a mean zonal spacing between like gridpoints in the Gulf of Alaska of about 7 km.

Realistic bathymetry and coastline geometry are included by using an NRLSSC improved version of the 1/12° ETOPO5 data set [National Oceanic and Atmospheric Administration (NOAA), 1986]. After interpolation to the model grid the bathymetry is filtered twice with a nine-point smoother to minimize energy generation at scales poorly resolved by the model [Leonardi, 1998]. The model coastline is chosen at the 200 m isobath, a typical value for the shelf break. This is done to obviate difficulties arising from the necessity that bottom topography be confined to the lowest layer. In addition, the
Plate 2. (a) Model upper layer thickness in the Gulf of Alaska on March 4, 1995. The black lines demarcate the southern and western boundaries of the study region. (b) Composite thermal image of the Gulf of Alaska from advanced very high resolution radiometer (AVHRR) data for March 1, 2, 3, and 10, 1995. The black lines demarcate the southern and western boundaries of the study region. From Thomson and Gower [1998].

amplitude of the bathymetry is scaled by 0.8 in order to ensure that no bottom features protrude from the lowest layer during the course of the integration.

Layer thicknesses and densities are chosen to be optimally consistent with the Levitus [1982] ocean climatology across the model domain (Table 1). Although these values are more than adequate to prevent layer surfacing or isopycnal outcropping (the diminution of layer thickness to zero) at tropical and subtropical latitudes, zero-layer thicknesses could occur in subarctic areas. The model solves this problem by invoking a “hydromixing” scheme when a layer depth $h_k$ falls below a predetermined entrainment thickness $h_e^*$. Mass and momentum from the layer below are entrained locally into the affected region of the surfacing layer. However, layer density values do not change, and in order to conserve mass in each layer, de-
elsewhere in the domain. See Wallcraft [1991] for a more detailed discussion.

Applied wind stress fields are the only forcing in this experiment. The hybrid European Centre for Medium-Range Weather Forecasts (ECMWF)/Helleraman and Rosenstein [1983] (HR) wind stress data set is used. It is constructed by removing the 1981–1995 climatological mean from pseudostresses derived from the 1000 mbar ECMWF 12 hourly winds and replacing it with that of the data set. The pseudo-stress fields are converted to stress fields by multiplication with the product of the drag coefficient, $C_D$, and the sea level density of air, $\rho_{atm}$.

Initial conditions are obtained by interpolating a snapshot of the oceanic state from a previous lower-resolution experiment onto the 1/16° grid. The experiment is then spun up to statistical equilibrium by forcing with the HR monthly mean wind stress climatology. After statistical equilibrium is reached, forcing begins on January 17, 1981, with the 12 hourly ECMWF/HR wind stress data set. The stress fields are linearly interpolated during calculation to the model time step of 12 min.

2.2. Model Response to Large-Scale Wind Forcing

Model-data comparisons using a variety of domain configurations and resolutions demonstrate that the NLOM produces realistic results for the Pacific Ocean north of 20.0°S [Donohue et al., 1994; Hogan et al., 1992; Hurlburt et al., 1992, 1996; Jacobs et al., 1994, 1996; Kindle and Phoebus, 1995; Melsom et al., 1999; Metzger et al., 1992, 1994; Metzger and Hurlburt, 1996; Mitchell et al., 1996; Mitchum, 1995]. Of particular note is comparison by Melsom et al. [1999] of sea surface height correlations at Sitka, Alaska. These show that good results can be obtained even in areas where isopycnal outcropping is expected.

The model reproduces all the major circulation features in and around the Gulf of Alaska (Plate 1). The zonal subarctic current is evident at about 45°N latitude. Its terminus lies at ~150°W longitude, where meridional coherence disintegrates into an alternating blend of powerful cyclonic and anticyclonic eddies. Along the eastern continental margin the Alaska Current ambles northwestward until it curves around the southern coast of Alaska and accelerates west of Kodiak Island, becoming the Alaskan Stream. The timing and position of recirculation filaments connecting the Alaskan Stream to the subarctic current vary according to the local wind stress curl [Thomson, 1972]. The interior of the Alaskan Gyre, which is defined by these boundaries, exhibits intense mesoscale activity in agreement with Cummins and Freeland [1993].

### Table 1. Parameters for NLOM Experiment P134-1.7

<table>
<thead>
<tr>
<th>Definition</th>
<th>Value</th>
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<tbody>
<tr>
<td>$C_b$</td>
<td>coefficient of bottom friction</td>
</tr>
<tr>
<td>$C_d$</td>
<td>drag coefficient</td>
</tr>
<tr>
<td>$C_M$</td>
<td>coefficient of additional interfacial friction associated with entrainment</td>
</tr>
<tr>
<td>$C_k$</td>
<td>coefficient of interfacial friction</td>
</tr>
<tr>
<td>$\theta_k$</td>
<td>acceleration due to gravity</td>
</tr>
<tr>
<td>$h_k$</td>
<td>km thickness at which detrainment starts</td>
</tr>
<tr>
<td>$\eta_k$</td>
<td>km thickness at which entrainment starts</td>
</tr>
<tr>
<td>$\bar{\omega}_k$</td>
<td>kth interface reference vertical mixing velocity</td>
</tr>
<tr>
<td>$\rho_{atm}$</td>
<td>sea level air density</td>
</tr>
<tr>
<td>$\rho_k$</td>
<td>density of layer k sigma $T$</td>
</tr>
<tr>
<td>$\Sigma_k H_k$</td>
<td>rest depth at base of layer k</td>
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<tr>
<td></td>
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3. Methodology

The experiment consists of four major steps. First, the analytical circulation equation is derived from the model equations of motion. This equation expresses vorticity balances as derived quantities of variables in the original equations of motion. Second, it is discretized in space and time in order to form an easily solvable algebraic equation. Third, gridded fields of model data from a single layer are used to calculate a value for each term in the algebraic equation. This is done for each data record, enabling time series of each term to be made. Finally, the scalar and field time series of the original and derived physical quantities are displayed visually in ways conducive to revealing the physics underlying interannual variability of the mechanisms governing the circulation in the Gulf of Alaska.

The horizontal domain of the experiment is the region defined by a western boundary situated at 205°E, a southern boundary at 50°N, and the adjoining British Columbia and Alaskan coasts (Plate 1). Quantitative evaluation of the spatial and temporal variability of vorticity along all potential boundaries indicates that these are an optimal compromise between the need to minimize stochastic influences along the periphery of the Gulf of Alaska and the desire to keep the domain as large as possible. Undesirable influences include the massive vorticity signature associated with the terminus of the Pacific Current and that associated with flow through the Unimak Pass.

A model layer within this domain can exchange vorticity through its western and southern boundaries and its upper and
The circulation equation is used to quantify the influence of these vorticity-transport pathways. This equation, derived directly from the model equations of motion, is for the topmost layer:

\[
C(t) - C(0) = \int_0^t \int_{\mathcal{B}^w} u(\zeta + f) \, dy \, dt + \int_0^t \int_{\mathcal{B}^v} v(\zeta) \, dy \, dt \\
+ A_w \int_0^t \int_{\mathcal{B}^k} \left( \nabla^2 \zeta \right) \, dA \, dt + (1 + C_M) \int_0^t \int_{\mathcal{B}^k} \left( k \cdot \nabla \right) w^* \, dA \, dt,
\]

where the subscript \( L \) denotes the next lower layer and \( w^* \) is the entrainment velocity through the bottom interface resulting from the model hydromixing process. (See Appendix A for details of the derivation of this circulation equation.)

The left-hand side of the equation is the circulation anomaly relative to the circulation at the start of the time integration. The first through fourth terms on the right-hand side are the contributions to the circulation from advection of vorticity across the western and southern boundaries, applied wind stress curl, and diffusion, respectively. The last term quantifies the change in circulation associated with vorticity input to a lower layer across the lower interface due to entrainment.

The time series of the circulation equation terms are computed from the discretized equation using the model \( u, v, w \), and \( h \) fields. All derivatives are first-order centered differences. Line and surface integrals use segments and rhomboidal areas proportioned for the poleward decrease of zonal grid spacing.

Each time series of the space integrals and the circulation are filtered with a 12 month running average to attenuate subannual variations. Integrating in time from the first filtered data record, which is 6 months into the raw model output record, then completes the evaluation of these terms. Various visualizations and statistical comparisons of these time series and the derived and native quantities with which they are constructed allow evaluation and quantification of the physical processes underlying interannual vorticity variability in the Gulf of Alaska.

4. Results

4.1. Atmospheric Forcing and Kelvin Waves

The Aleutian Low is the dominant atmospheric feature in the northeast Pacific Ocean and is the driving force behind the cyclonic mean oceanic circulation in the Gulf of Alaska. Emery and Hamilton [1985] suggest that weak winter atmospheric circulation in the North Pacific often precedes major El Niño events and is followed by a rapid strengthening of the circulation pattern. This sequence is verified in observations analyzed by Putman [1998]. Changes in the spatial coverage of the low, however, yield a different pattern to the variability in a limited domain, such as that currently under study. For example, even if the atmospheric low deepens, increasing the wind stress curl, the curl of the wind away from the low center can be of opposite sign to the curl of the wind at the low center. This local change in wind stress curl sign can reduce the always positive area-integrated wind stress curl over the Gulf of Alaska study region.

The area-integrated curl of the wind stress divided by the ocean upper layer thickness, i.e., the time rate of change of circulation of the ocean by the wind (Figure 1), clearly shows interannual variability associated with ENSO (Figure 2) but with a pattern out of phase with that expected for the North Pacific as a whole.

Tropical Pacific cyclones are a major source of the equatorial Kelvin waves associated with El Niño events [Kindle and Phoebus, 1995]. An eastward moving equatorial Kelvin wave reflects off the eastern boundary in the form of a westward traveling Rossby wave and two poleward traveling coastal Kelvin waves [Clarke, 1983]. Meyers et al. [1998], extending work by Chelton and Davis [1982] and Enfield and Allen [1980], use coastal sea level data along the northeast Pacific Ocean to verify that coastal Kelvin waves propagate from the equator to the northern Gulf of Alaska. Recent modeling studies by Melson et al. [1999] show that El Niño-induced coastal Kelvin waves can alter circulation in the Gulf of Alaska by destabilizing the Alaska Current, spawning coast-wide trains of long-lived anticyclonic eddies. In the present study, four of the five greatest maxima in the coastal upper layer thickness time series at 50°N (Figure 3) occur during the El Niño years 1982–1983, 1986–1987, 1991–1992, and 1994–1995, which was nearly defined as an El Niño year by the Japanese Meteorological Agency (JMA) index (Figure 2). The fifth occurs during the

![Figure 2. JMA Index: an index of the equatorial Pacific sea surface temperature from the Japan Meteorological Agency.](image-url)
months preceding the 1986–1987 El Niño, which was a period of downwelling Kelvin wave formation associated with the rapid transition to El Niño conditions. Each event is characterized by a positive anomaly in the coastal upper layer thickness near the wintertime peak in the annual cycle (Figure 4). Furthermore, each is preceded by a characteristic sharp increase in the anomaly time series. These distinct increases in the coastal upper layer thickness do not appear to be correlated with changes in the space-integrated wind stress curl, which is increasing during the 1985–1986 and 1986–1987 upper layer thickness maxima and decreasing during the 1982–1983, 1991–1992, and 1994–1995 maxima. This disparity precludes the possibility that the maxima are solely due to increased Ekman pumping from an intensified atmospheric circulation pattern. The sharp increases in upper layer thickness are therefore likely due, at least in part, to remotely forced downwelling

Figure 3. Coastal upper layer thickness at 50°N from late 1981 to 1995. The arbitrary bar is set to 111 m.

Figure 4. Time rate of change circulation from the wind stress curl (prior to filtering) plotted with coastal upper layer thickness anomaly from the climatology at 50°N from 1982 to 1995. The first segment is the upper layer thickness annual climatology.

4.2. Eddy Train Genesis and Fate

Coast-wide eddy trains whose diameter and spacing are consistent with generation by baroclinic instability processes have been modeled by Melsom et al. [1999] and observed by Thomson and Gower [1998] and Crawford and Whitney [1999]. In this experiment, eddy activity is revealed in the model upper layer thickness throughout the record. Eddy trains consisting of at least small and short-lived eddies appear in every year except those following the 1988–1989 La Niña. This model result strongly supports Thomson and Gower’s [1998] hypothesis of the existence of an annual cycle of eddy generation. The absence of eddy trains following the 1988–1989 La Niña is suggestive of a link between the annual cycle and the ENSO and is consistent with altimetry observations by Meyers and Basu [1999]. Whether intraseasonal coastal Kelvin waves not related to El Niño or perturbations in the local wind stress field drive the annual cycle is unknown.

Coast-wide trains of robust mesoscale eddies appear five times in the model record: 1983, 1986, 1987, 1992, and 1995. The number and spacing of the 1995 eddy train show close agreement to the contemporaneous composite thermal image discussed by Thomson and Gower [1998] (Plate 2). The eddies observed in these 5 years are sharply differentiated from those of other years by their size and longevity. They can exceed 200 km diameter and last more than a year, whereas the smaller eddies never exceed 80 km and rarely last more than 2 months. All three El Niño events in the model record contain at least one large-eddy train. The eddy train formation in 1995 occurred during a year that was nearly classified as an El Niño year, and the 1986 eddy train was associated with the rapid transition to an El Niño event. Examination of the 50°N coastal upper layer thickness anomaly time series reveals the most incriminating observation: all of the large-eddy trains appear immediately following relative maxima that are concurrent with the peak in the annual cycle. These maxima have been shown in section 4.1 to be linked with tropically forced coastal Kelvin waves (Figure 4).

Increase in the geostrophic flow of the Alaska Current caused by powerful tropically forced coastal Kelvin waves excited during El Niño events is therefore the critical component of a large-eddy train genesis event. Simultaneous increases in the mean flow caused by wind stress–induced Ekman pumping no more than enhance the size of an event. However, local reversals in wind stress, as hypothesized by Thomson and Gower [1998], may compete with the later Kelvin waves of a packet as the source of perturbation necessary to induce baroclinic instability.

Size and latitude of eddy origination in part determine fate. Small eddies are either quickly reabsorbed into the boundary current or tend to dissipate within 2 months. Large eddies that form near the head of the gulf are inevitably drawn into the Alaskan Stream. Those that form farther south along the coast tend to propagate to the southwest. Some of these appear to be arrested in the center of the gyre, while others halt upon reaching the Pacific Current front. Those that reach the front have carried negative vorticity to the periphery of the Gulf of Alaska.

Three types of eddy train generation patterns appear to be present in the model record. No eddy trains appear during La Niña events. During neutral phases, coast-wide trains of small, short-lived eddies emerge, usually around the peak of the annual cycle of the Alaska Current. These weaker events may be

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Table 2. Statistics for Circulation Equation Time Integral Integrands

<table>
<thead>
<tr>
<th>Term</th>
<th>Variance, m² s⁻²</th>
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</thead>
<tbody>
<tr>
<td>(dC/dt) : WS(t)</td>
<td>0.79</td>
</tr>
<tr>
<td>WS(t) : SB(t)</td>
<td>-0.68</td>
</tr>
<tr>
<td>WB(t) : SB(t)</td>
<td>-0.25</td>
</tr>
<tr>
<td>WS(t) : WB(t)</td>
<td>-0.83</td>
</tr>
<tr>
<td>WS(t) + SB(t) : WB(t)</td>
<td></td>
</tr>
</tbody>
</table>

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**Note:** The table includes terms from the circulation equation and their corresponding variances, which are crucial for understanding the variability and balance of the system.
caused by intraseasonal coastal Kelvin waves or wind stress reversals. Coast-wide trains of large, long-lived eddies occur only following periods of remote generation of downwelling Kelvin waves, which reach the Alaskan Current at the proper time. Only these latter eddies have the size and longevity necessary to serve as conduits for consequential negative vorticity advection out of the Gulf of Alaska.

4.3. Vorticity Balances

The wind stress curl and vorticity advection through the southern and western boundaries are the primary governing mechanisms for the circulation in the Gulf of Alaska. The variance of the time rate of change of circulation in the gulf is an order of magnitude smaller than the variance of the time rate of the circulation attributed to each of these three most dominant components alone (Table 2). The interannual variations in these sources and sinks of vorticity must therefore at least partially balance to prevent a wildly varying circulation in the domain.

The wind stress curl is the largest vorticity source of the upper layer of the ocean. Although weaker in magnitude, interannual variations in the vorticity advection through the southern boundary tend partially to balance interannual variations in the wind stress curl (Figure 5). The relationship is quantified by a linear correlation coefficient of \(-0.68\). The interannual variability in the vorticity added by wind stress curl is largely associated with the Southern Oscillation. The interannual variability of vorticity advection across the southern boundary is primarily caused by the passage of anticyclonic eddies. All of the large relative highs in the southern boundary time series are associated with the passage of a single eddy.

A Hovmöller diagram of the upper layer thickness verifies this effect of the eddies on the southern boundary vorticity advection by revealing the time and longitude of eddy transits across the boundary (Plate 3). Positive thickness anomalies indicate that the eddies are anticyclonic and therefore embody negative vorticity. The zonal direction of propagation is revealed to be westward by the negative slope of the layer thickness signature of eddies in the western half of the domain. However, this plot contains no information on the meridional direction of propagation; animations of the upper layer thickness reveal it to be uniformly southward.

Vorticity advection across the western boundary is the primary sink of the upper layer (Figure 6a). The predominant physical mechanism is the continual westward advection of positive vorticity by the Alaskan Stream (Figure 6b). The negative correlation \((r = -0.83)\) between the western boundary term and the sum of the curl and southern boundary terms suggests that interannual variation of vorticity advection through the western boundary is driven by variability in the latter terms. Therefore it appears that the Alaskan Stream serves as a "vorticity overflow valve" responsive on interannual timescales and compensating for imbalances wrought by the driving terms.

Diffusion is a secondary vorticity sink. It is negatively correlated with circulation (Figure 7). Its role in the vorticity balance is minor because it is two orders of magnitude smaller than the dominating terms of the circulation equation.

Entrainment of vorticity to the upper layer has a small positive contribution to the circulation in the Gulf of Alaska. Its existence is due to bydromixing in the NLOM, where mass and momentum from the underlying layer are added at \(h\) grid points whose \(h\) values fall below a predetermined level. This term is a consequence of the model's not allowing a layer's thickness to reach zero. It may or may not occur in the physical world. Nevertheless, its contribution to the interannual variability of the circulation is an order of magnitude smaller than the other terms (less diffusion).

The balances of the vorticity sources and sinks as described result in a circulation varying with time slower than the driving mechanism would otherwise dictate. The time rate of change of circulation is positively correlated with the driving wind
stress ($r = 0.79$), but vorticity sinks, primarily vorticity advection out of the domain, serve to balance the vorticity input. This has the effect of reducing the interannual variability of the circulation (Figure 8).

The time-integrated terms in the circulation equation represent the cumulative effect on the circulation anomaly in the Gulf of Alaska of vorticity flow through the various pathways. Visual inspection of these terms can best illuminate the relative importance of their variability in the maintenance of the circulation (Figure 8). Illustrated are the cumulative effects of interannual variability during a period of robust ENSO activity on the vorticity balances in the Gulf of Alaska. The time-integrated wind stress curl shows the reduction in vorticity input in 1983 superimposed upon an essentially linear increase in its contribution to the circulation. The time-integrated southern boundary term shows a positive slope in 1983 resulting from the transit of a large eddy spawned during the 1982-1983 El Niño. The entrainment term is nearly linear, and the diffusion term is too small to be shown, leaving interannual variability in the time-integrated western boundary term to balance variations in the sum of the time-integrated curl and southern boundary terms. The other two distinct ENSO events in the model data record display similar patterns.

5. Summary and Conclusions

The interannual variability of the upper ocean vorticity budget in the Gulf of Alaska has been explained with the aid of a high-resolution layered ocean model. The circulation equation, which expresses the balances as derived quantities of model variables, was derived directly from the model equations of motion. Each term in the equation represents a different pathway through which vorticity can flow. The terms of the equation were discretized and computed from the gridded model output fields, and a time series of the solution of each term was constructed.

Examination of these time series demonstrates that the interannual variability in vorticity balances is predominantly due to atmospheric and oceanic teleconnections originating with ENSO. The input of vorticity to the upper ocean by the wind stress curl exhibits the strongest variability. This is partially balanced by the advection of vorticity through the southern boundary. Analysis of the time-varying model fields demonstrates that this occurs when large anticyclonic eddies migrate southward out of the region. Further evidence of the generating mechanism for these eddies has also been presented. The formation of the large eddies is not always coincident with an increase of the local wind stress curl but is correlated with the arrival of downwelling coastal Kelvin waves originating in the equatorial Pacific. When these Kelvin waves reach the Alaskan Current at the peak of the annual cycle, the current is strengthened, triggering baroclinic instabilities that result in the formation of a coast-wide train of anticyclonic eddies [Melsom et al., 1999]. The more southerly of these eddies propagate to the southwest, carrying their associated negative vorticity out of the Gulf of Alaska. Negative vorticity advection in the Alaskan Stream contributes further to the balance. Though the wind stress curl is the driving mechanism for the circulation in the domain, the nonlinear processes of vorticity advection associated with eddy propagation to the south and the Alaskan Stream to the west contribute to the circulation in a way to reduce its interannual variability.

Appendix A

Consider the equations of motion for the upper layer of an n-layer finite depth model:
Plate 3. Hovmöller diagram of upper layer thickness along the southern boundary at 50°N

\[
\frac{\partial \mathbf{v}}{\partial t} + (\nabla \cdot \mathbf{v} + \mathbf{v} \cdot \nabla)\mathbf{v} + \mathbf{k} \times \mathbf{v} + \mathbf{k} \times \mathbf{f} = -h \sum_{i=1}^{n} G_{i} \nabla (h_{i} - H_{i}) + \frac{\tau_{w}}{\rho_{0}} + A_{sh} \nabla^{2} \mathbf{v} + \max (0, w_{s}) \mathbf{v}_{L} - \max (0, -w_{s}) \mathbf{v} + C_{m} \max (0, w_{s}) (\mathbf{v}_{L} - \mathbf{v}) - w_{s} \mathbf{v}.
\]

Expand the second term using the vector identity \((\mathbf{v} \cdot \nabla)\mathbf{v} = \frac{1}{2} \nabla^{2} \mathbf{v} - \mathbf{v} \times \nabla \times \mathbf{v}:

\[
h \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{2} \nabla^{2} \mathbf{v} - \mathbf{v} \times \nabla \times \mathbf{v} + \mathbf{k} \times \mathbf{f} + \frac{1}{2} \nabla^{2} \mathbf{v}
\]

\[
\times \nabla \times \mathbf{v} + \mathbf{k} \times \mathbf{f} = -h \sum_{i=1}^{n} G_{i} \nabla (h_{i} - H_{i}) + \frac{\tau_{w}}{\rho_{0}} + A_{sh} \nabla^{2} \mathbf{v} + \max (0, w_{s}) (\mathbf{v}_{L} - \mathbf{v}) - w_{s} \mathbf{v}.
\]

Divide by \(h\) and take the curl:

\[
\frac{\partial}{\partial t} (\nabla \times \mathbf{v}) + \nabla \times \frac{1}{2} \nabla^{2} \mathbf{v} - \nabla \times \mathbf{v} \times \nabla \times \mathbf{v} + \nabla \times \mathbf{h}^{-1} (k \mathbf{k} \times \mathbf{f} \mathbf{v}) = \nabla \times - \sum_{i=1}^{n} G_{i} \nabla (h_{i} - H_{i}) + \nabla \times \frac{\tau_{w}}{\rho_{0} h} \nabla
\]

\[
\times A_{sh} \nabla^{2} \mathbf{v} + r,
\]

where the subscript \(L\) denotes the next lower layer and \(w_{s} = w_{k} (for k = 1)\) is the entrainment velocity through the bottom interface resulting from the model hydromixing process. Expand the first term and simplify using the continuity equation:
where

\[ r = \nabla \times \left( \frac{1}{h} \left[ \max(0, w_*) \nabla L - \max(0, -w_*) \nabla \right] + C_M \max(0, w_*) (\nabla L - \nabla - w_*) \right). \]

The second and fifth terms vanish because \( \nabla \times \nabla \phi = 0 \), where \( \phi \) is any scalar:

\[ \frac{\partial}{\partial t} (\nabla \times v) - \nabla \times \nabla \times v + \nabla \times h^{-1}(k \times fV) = \nabla \times \frac{\tau_w}{\rho \beta h} + \nabla \times A_{\nu} \nabla^2 v + r. \]

Use the vector identity \( \nabla \times (A \times B) = (B \cdot \nabla)A - (A \cdot \nabla)B + A(\nabla \cdot B) \) to expand the second term, where \( A = v \) and \( B = \nabla \times v = \zeta = \zeta' \):

\[ \frac{\partial \zeta}{\partial t} - \left[ (\zeta' \cdot \nabla) - (\nabla \cdot \zeta') \right] + \nabla \times h^{-1}(k \times fV) = \nabla \times \frac{\tau_w}{\rho \beta h} + \nabla \times A_{\nu} \nabla^2 v + r. \]

The first and fourth terms in brackets are zero because \( \nabla \) is a horizontal operator:

\[ \frac{\partial \zeta}{\partial t} + \left[ (\zeta' \cdot \nabla) - (\nabla \cdot \zeta') \right] + \nabla \times h^{-1}(k \times fV) = \nabla \times \frac{\tau_w}{\rho \beta h} + \nabla \times A_{\nu} \nabla^2 v + r. \]

Next expand the Coriolis term and simplify:

\[ \nabla \times h^{-1}(k \times fV) = \nabla \left[ \frac{\partial}{\partial x} (fu) + \frac{\partial}{\partial y} (fv) \right] = \nabla (f \cdot v). \]

Expand the result using the vector identity \( \nabla \cdot cA = c(\nabla \cdot A) + A \cdot \nabla c \), where \( c \) is any scalar and \( A \) is any vector:

\[ \nabla \times h^{-1}(k \times fV) = [f(\nabla \cdot v) + v \cdot \nabla f]k. \]

Next expand the diffusion term:

\[ \nabla \times A_{\nu} \nabla^2 u = A_{\nu} \nabla \times (\nabla^2 u) = kA_{\nu} \left( \frac{\partial}{\partial x} \nabla^2 u \right). \]

Substitute (A2) and (A3) into (A1):

\[ \frac{\partial \zeta}{\partial t} + [\zeta(\nabla \cdot v) + (v \cdot \nabla) \zeta] + f(\nabla \cdot v) + (v \cdot \nabla) f \zeta = \nabla \times \frac{\tau_w}{\rho \beta h} + k A_{\nu} (\nabla^2 \zeta) + r. \]

Combine terms 2 with 4 and 3 with 5:

\[ \frac{\partial \zeta}{\partial t} + (\nabla \cdot v) + (v \cdot \nabla) (\zeta + f) = \nabla \times \frac{\tau_w}{\rho \beta h} + A_{\nu} (\nabla^2 \zeta) + \nabla \times \frac{\tau_w}{\rho \beta h} + k A_{\nu} (\nabla^2 \zeta) + r. \]

Now, consider \( r \), the terms due to diapycnal mixing or entrainment:

\[ r = \nabla \times \frac{1}{h} \left[ \max(0, w_*) \nabla L - \max(0, -w_*) \nabla \right] + C_M \max(0, w_*) (\nabla L - \nabla - w_*) \]

There is no local detrainment in this model experiment (see Table 1). Neglecting the spatially uniform detrainment velocity associated with the global mixing balance term, \( w_* \) will be always nonnegative. Thus it is proper to write

\[ r = (1 + C_M) \nabla \times \frac{w_*}{h} (\nabla L - \nabla), \]

where

\[ w_\ast = \frac{w_k}{4} \left( \frac{h}{h - 1} - 1 \right) \quad k = 1 \]

(see notation). Take the surface integral of (A4), where \( D \) is the domain of integration:

\[ \int_D (A4) \cdot dA = \int_D (A4) \cdot \hat{n} \, dA = \int_D (A4) \cdot k \, dA \frac{\partial C}{\partial t} \]

\[ = - \int_D (\nabla \cdot v + v \cdot \nabla) (\zeta + f) \, dA + \int_D (\zeta \cdot v) \]

\[ \times \frac{\tau_w}{\rho \beta h} \, dA + A_{\nu} \int_D (\nabla^2 \zeta) \, dA + (1 + C_M) \int_D \left( \zeta \cdot v \right) \]

\[ \times \frac{w_*}{h} (\nabla L - \nabla) \, dA \]

Using \( \int_D (\nabla \times v) \cdot dA = \int_{\partial D} v \cdot ds = C \), where \( C \) is the circulation. Simplify the first term on the right-hand side:

\[ \int_D (\nabla \cdot v + v \cdot \nabla) (\zeta + f) \, dA = - \int_D \left( u_x + u_y + u \frac{\partial}{\partial x} \right) \]

\[ + \frac{\partial}{\partial y} \left( \zeta + f \right) \, dA = - \int_D \left\{ \frac{\partial}{\partial x} \left[ u(\zeta + f) \right] + \frac{\partial}{\partial y} \left[ u(\zeta + f) \right] \right\} \, dA \]

\[ + u(\zeta + f) \, dy - u(\zeta + f) \, dx \]

Let the domain \( D \) be a box approximating the Gulf of Alaska, with closed boundaries on the northern and eastern sides. Carry out the line integral counterclockwise from the northwest vertex. Integration limits will be denoted by NW for the northwest vertex of the box, etc.:

\[ - \int_D (\nabla \cdot v + v \cdot \nabla) (\zeta + f) \, dA = - \int_{\text{NW}} u(\zeta + f) \, dy \]

\[ + \int_{\text{SW}} v(\zeta + f) \, dx - \int_{\text{NE}} u(\zeta + f) \, dy + \int_{\text{NW}} v(x + f) \, dx \]

The third and fourth terms on the right-hand side vanish with the kinematic boundary condition:
\[- \int_D (\nabla \cdot \mathbf{v} + \mathbf{v} \cdot \nabla) (\zeta + f) \, dA = \int_{\text{sw}} u(\zeta + f) \, dy \]
\[+ \int_{\text{sw}} v(\zeta + f) \, dx.\]

Integrate in time from 0 to \(t\) to yield the circulation equation:

\[\frac{dC}{dt} = \int_{\text{sw}} u(\zeta + f) \, dy + \int_{\text{sw}} v(\zeta + f) \, dx + \int_D \left( k \cdot \nabla \right) \, dA + A_M \int_D \left( \nabla^2 \zeta \right) \, dA + (1 + C_M) \int_D \left( k \cdot \nabla \right) \, dA \]
\[\times \frac{\tau_w}{\rho_0} \, dA + \frac{w_e}{h} (\mathbf{v}_L - \mathbf{v}) \, dA.\]

Substitute into (A6):

\[\frac{\partial C}{\partial t} = \int_{\text{sw}} u(\zeta + f) \, dy + \int_{\text{sw}} v(\zeta + f) \, dx + \int_D \left( k \cdot \nabla \right) \, dA + A_M \int_D \left( \nabla^2 \zeta \right) \, dA + (1 + C_M) \int_D \left( k \cdot \nabla \right) \, dA \]
\[\times \frac{\tau_w}{\rho_0} \, dA + \frac{w_e}{h} (\mathbf{v}_L - \mathbf{v}) \, dA.\]

Integrate in time from 0 to \(t\) to yield the circulation equation:

\[C(t) - C(0) = \int_0^t \int_{\text{sw}} r(\zeta + f) \, dy \, dt + \int_0^t \int_{\text{sw}} \left( \nabla \cdot \mathbf{v} \right) \, dx \, dt + \int_0^t \int_D \left( \nabla \cdot \mathbf{v} \right) \, dA \, dt \]
\[+ A_M \int_0^t \int_D \left( \nabla^2 \zeta \right) \, dA \, dt + (1 + C_M) \int_0^t \int_D \left( \nabla \cdot \mathbf{v} \right) \, dA \, dt \]
\[\times \frac{\tau_w}{\rho_0} \, dA + \frac{w_e}{h} (\mathbf{v}_L - \mathbf{v}) \, dA \, dt.\]

Notation:

- \(A_H\) coefficient of isopycnal eddy viscosity.
- \(C(0)\) circulation at the first data record.
- \(C(t)\) circulation.
- \(C_k\) coefficient of interfacial friction.
- \(C_b\) coefficient of bottom friction.
- \(C_M\) coefficient of additional interfacial friction associated with entrainment.
- \(D(x, y)\) total ocean depth at rest.
- \(f\) Coriolis parameter.
- \(g\) acceleration due to gravity.
- \(G_H = \frac{f}{g}\) for \(l = k\).
- \(G_k = \frac{g}{l(l - 1)(\rho_0 - \rho_l)}\) for \(l < k\).
- \(h_k\) \(k\)th layer thickness.
- \(h_k^+\) \(k\)th layer thickness at which entrainment starts.
- \(h_k^-\) \(k\)th layer thickness at which detrainment starts.
- \(H_k\) \(k\)th layer thickness at rest.
- \(H_n = D(x, y) - \sum_{i=1}^{n-1} H_i\) for \(n\)th layer thickness at rest.
- \(f, j, k\) zonal, meridional, and vertical unit vectors.
- \(n\) number of isopycnal layers.
- \(t\) time.
- \(v_k\) \(k\)th layer velocity.
- \(V_k = h_k v_k\) for \(k\)th layer transport.
- \(\rho_k\) \(k\)th layer density, constant in space and time.
- \(\rho_0\) reference density, constant in space and time.
- \(\tau_w\) wind stress.
- \(\tau_k = \tau_w\) for \(k = 0\).
- \(\tau_k = C_k \rho_0 (v_k - v_{k+1})(v_k - v_{k+1})\), for \(k = 1 \ldots n - 1\).
- \(\tau_k = C_k \rho_0 v_{in}\) for \(k = n\).
- \(\omega_k = 0\) for \(k = 0, n\).
- \(\omega_k = \max (0, \omega_k) - \max (0, \omega_k) - h_k \omega_k\) for \(k = 1 \ldots n - 1\).
- \(\omega_k^+ = \omega_k h_k^+/4[(1/h_k^-) - (1/h_k^+)]\).
- \(\omega_k^- = \omega_k h_k^-/4[(1/h_k^-) + h_k^- - h_k^- - (1/h_k^-)]\).
- \(\omega_k = \{\int [\max (0, \omega_k) - \max (0, \omega_k)]/1 \int H_k\) global mixing balance term.
- \(\omega_k\) \(k\)th level interface reference vertical mixing velocity.
- \(\zeta\) vorticity, \(\nabla \times \mathbf{u}\).

References:


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