An Investigation of the Diabatic Wind Profile of the Atmospheric Boundary Layer

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Abstract. Over 2800 mean wind profiles (data from two mobile micrometeorological stations near Dallas, Texas) were analyzed to test the Monin-Obukhov similarity hypothesis as expressed by the log-linear wind profile. Wind speed and temperature data for eight levels between 25 cm and 32 m were averaged for 5- and 25-min periods. The Monin-Obukhov parameter $\alpha'$, which is the coefficient of the linear term in the profile, was found to be a function of stability varying from 0.2 to 3.0 in lapse conditions and from 9 to 3 during inversion conditions. However, practical values of $\alpha'$ for lapse (0.6) and inversions (5.0) conditions are suggested for forecasting mean diabatic wind profiles. The Obukhov 'gradient length' $L'$ was found to be a function of height above the ground; therefore, it does not possess complete dynamic similarity. The broad applicability of the log-linear profile is shown by calculating residual variances for each profile as a function of height, wind speed, and stability. An empirical log-power profile was also tested. The preferred power for the diabatic term was unity (or larger) for inversion conditions and approximately 0.5 for lapse conditions. Since the standard error of estimate of the wind speed based on the log-linear profiles was not significantly greater than that based on log-power for the latter case, the log-linear profile appears to be adequate for all stability regimes.

It is accepted generally that the logarithmic wind profile satisfactorily predicts the relationship between wind speed and shearing stress in a fully turbulent boundary layer under neutral (adiabatic) conditions. The equations

$$\frac{\partial u}{\partial z} = \frac{U_*}{kz} \quad (1)$$

and

$$u^2 = \left( \frac{U_*}{k} \right) \ln \left( \frac{z}{z_0} \right) \quad (2)$$

uniquely describe the shape of the neutral wind profile. The notation is standard, and the symbols are defined in the appendix. Sutton [1953], Fleagle and Businger [1963], and others derive these relationships. The most critical assumptions implied by (1) and (2), which should be practically met in an analysis of observations, are the conditions of steady-state, horizontal uniformity, and the constancy of shearing stress with height.

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1 This paper is an abridgement of the author’s M.S. thesis; contribution of the Department of Oceanography and Meteorology, Texas A&M University.
The logarithmic profile is strictly applicable only when neutral (near-adiabatic) thermal stratification dominates the atmospheric boundary layer. During lapse conditions (greater than adiabatic), $\partial u / \partial z < \ln z$. During inversion conditions (less than adiabatic), $\partial u / \partial z > \ln z$. The wind profiles associated with nonadiabatic temperature distributions are called diabatic. Numerous theories have been advanced to explain the variation of wind shear due to thermal stratification. Elliott [1957] pointed out the equivalence of the various approaches. In the present work the accepted theory is the well-known Monin and Obukhov [1954] log-linear profile; this paper will be referred to as MO.

In the subsequent paragraphs, the MO hypothesis and the present observational evidence will be briefly discussed.

Consider the logarithmic wind shear,

$$ S = k z U_*^{-1} \partial u / \partial z = f(\xi) $$

which depends only on a stability parameter $\xi$. Physically, for the neutral wind profile to be valid, $S$ must be unity in a neutral (isentropic) atmosphere and will approach unity close to the ground in any atmosphere. Following Obukhov [1946], MO postulated from dynamic similarity concepts that a universal relationship exists between $S$ and $\xi = z/L$ which fully characterizes thermal effects on the wind profile; $L$ is the Obukhov scale length, defined by

$$ L = - U_* s C_s \theta / k g H $$

where $H$ is positive upward and proportional to $-\partial \theta / \partial z$. Thus, $L$ is positive for stable (inversion) conditions and negative for unstable (superadiabatic) conditions.

Since dimensional analysis cannot be used to determine the specific function, the usual technique is to expand $S(z/L)$ in a power series, take a linear approximation, integrate over the range $z_0$ to $z$, and thus obtain the log-linear profile,

$$ u = U_* k^{-1} [\ln (z/z_0) + \alpha(z - z_0)/L] \quad z > z_0 $$

Since turbulent heat flux is rarely measured directly, many investigators introduce the ‘gradient length’, $L'$, defined by

$$ L' = (K_H/K_M) \quad L = U_* \theta (\partial u / \partial z) / kg (\partial \theta / \partial z) $$

If we introduce a new coefficient, $\alpha' = \alpha K_H/K_M$, (5) becomes

$$ u = U_* k^{-1} [\ln (z/z_0) + \alpha'(z - z_0)/L'] $$

The parameter $\alpha'$ is to be determined from observations. In the present study the practical validity of (7) is investigated. Until extensive measurements of $H$ are available, it is necessary to use $K_H/K_M$, although it is recognized that important issues have been set aside upon the introduction of this ratio.

An alternative solution for $S$ has been derived in different ways by Kazanski and Monin [1956], Ellison [1957], Yamamoto [1959], Panofsky [1961], and Sellers [1962]. By the use of limiting conditions for free and forced convection, we obtain

$$ S^4 - \eta S^3 / L' = 1 $$

For small $z/L'$, (8) is equivalent to (7), where $\alpha' = n/4$. Ellison emphasizes that (8) has no theoretical basis beyond having the correct form in limiting conditions.

Monin and Obukhov interpreted $\alpha'$ as a universal wind profile constant and found $\alpha' = 0.6$. Ellison derived $\alpha' = 0.7$. On the other hand, Panofsky et al. [1960] chose $\alpha'$ to be about 4.5 on the basis of data from several sources. Taylor [1960] reanalyzed MO data and also data from Swinbank [1955] and stressed that the determination of $\alpha'$ is critically dependent on the range of $z/L$ chosen. Taylor concluded that $\alpha = 6.1$. Also, recent work by Japanese investigators indicates that $\alpha'$ may vary with stability, having values of the order of 5 to 10 under stable regimes and 1 to 5 under unstable regimes [e.g., Yamamoto, 1959; Takeuchi, 1961; Shiotani, 1962]. It is evident that, although the log-linear profile is generally accepted, there is considerable doubt regarding the parameter $\alpha'$ and the range of applicability.

In the light of these questions, the primary objectives of this work were (1) to determine whether $\alpha'$ is a universal constant in all stability regimes; (2) to determine the range of applicability of the log-linear profile; and (3) to investigate other empirical forms of the diabatic wind profile. A rather comprehensive set of micrometeorological data, not previously available, was used.
Clearly, the sites are not as ideal as those used during the two expeditions to O’Neill, Nebraska, in 1953 and 1956.

The design of the stations was based primarily on the specific requirements of micrometeorological research. In normal operation the following parameters were measured at each station.

1. Temperature: wet- and dry-bulb temperatures at \( \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, \) and \( 32 \) m elevation. Soil temperatures at \(-3, -6, -12, -25, -40, -65, \) and \(-100 \) cm.
2. Wind speed: at the same elevations used for air temperature determinations.
3. Wind direction: at any one level between the surface and \( 32 \) m; for this study at \( 6 \) m.
4. Soil heat flux: at any one level between \( 0 \) and \(-1 \) m.
5. Incoming solar radiation.
6. Albedo.

The electronic details of design will not be discussed. It is sufficient to consider that all...

DATA AND PROCEDURE

The extensive micrometeorological data of the Dallas Tower Project which Texas A&M University has operated in the Cedar Hill area were made available to me. This project had as its basic objective the recording of micrometeorological data in sufficient quantity and quality to test a low-level (2 m to 1050 m) meteorological simulator model on an analog computer. A detailed description of the site and the measuring and data collection equipment is given in a research report by Clayton and Eckelkamp [1961]. Data from the two automatic (or main) stations were chosen for study. These stations, called stations A and B, are located 5 miles apart in gently rolling farmland near Dallas, Texas. Figure 1 illustrates the 32-m towers used at both stations and the topography southeast from station A toward a television tower. The short hay and lower sensors at station B can be seen in Figure 2.

It is assumed that these sites will meet the model requirement of horizontal uniformity.
signals from the sensors proceeded to the analog memory and thence to the readout system without modification to the signals other than amplification and conversion to digital mode. The readout system consisted of a digital voltmeter and a typewriter (IBM output-writer) and punched paper tape.

For this study the wind speed and dry-bulb temperature measurements at the eight heights from 25 cm to 32 m and the wind direction at 6 m were used. Temperatures were sensed by copper-constantan thermocouples (single junction) referenced against 40.0 ± 0.003°C. The temperature was recorded once each minute, 4 out of 5 minutes, and is considered reliable to 0.05°C.

Wind speeds (4-minute averages in cm/sec) were measured by photoelectric anemometers with electromechanical counters. In each fifth minute the readout system recorded the averaged wind speed for all eight levels. Above the threshold speed of the anemometers (20 cm/sec), the wind speed data are considered reliable to 3%.

The wind direction sensor was a standard electrical wind vane with the 5° dead zone located to the west, a direction from which the wind seldom blows. The wind direction from north was recorded once each minute, 4 out of 5 minutes, from −85 to 270°. Although this signal was read to 1/10°, the wind direction values are considered reliable to within 5°.

From the 1½ years' data available, a large but manageable sample was chosen more or less randomly. The criteria for selection were (1) favorable weather conditions, i.e., air mass conditions at the station during the chosen period; (2) all required data available; (3) winter and summer cases; and (4) day and night cases.

Since these criteria are independent of the fine structure of the wind profile, the sample is considered unbiased within the objectives. Admittedly, these criteria do not guarantee the steady-state assumption; however, it is believed that a more subjective analysis to select cases would have biased the sample. The raw data available on punched cards were time-averaged in 5-min arithmetic averages and 25-min triangular-weighted averages and stored on magnetic tape for further analysis. Table 1 summarizes the temporal distribution of the 2862 five-minute cases and the associated 477 twenty-five-minute cases. Since the station may be used for special experiments, especially in the daytime, and no emphasis was placed in this study upon having the same number of cases for each month, the distribution of cases from month to month was not uniform.

There appears to be no proved criterion for determining the minimum time-averaging period for mean wind profile studies. It is known that time-averaging over a given interval, \( \psi \), filters out the high-frequency components with periods much smaller than \( \psi \) and retains the low-frequency components with periods larger than \( \psi \) [e.g., Monin, 1958]. It is desirable to select \( \psi \) so

<table>
<thead>
<tr>
<th>TABLE 1. Data—Temporal Distribution of Five-Minute Wind Profiles (Entries are numbers of cases in each 4-hour period for selected months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local Dallas Time</td>
</tr>
<tr>
<td>0000-0400</td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Station A</strong></td>
</tr>
<tr>
<td>July 1962</td>
</tr>
<tr>
<td>Aug. 1962</td>
</tr>
<tr>
<td>Sept. 1962</td>
</tr>
<tr>
<td>Oct. 1962</td>
</tr>
<tr>
<td>Jan. 1963</td>
</tr>
<tr>
<td>Feb. 1963</td>
</tr>
<tr>
<td><strong>Station B</strong></td>
</tr>
<tr>
<td>June 1962</td>
</tr>
<tr>
<td>July 1962</td>
</tr>
<tr>
<td>Feb. 1963</td>
</tr>
</tbody>
</table>
DIABATIC WIND PROFILE OF THE BOUNDARY LAYER

that the contribution to the fluxes from the turbulent elements containing the main part of the turbulent energy are included in the average. Monin [1958] suggests a time interval of the order of 10 minutes; Swinbank [1955] suggests 5 minutes. Although many authors do not specifically disclose the averaging period, it is believed that intervals of 5 to 60 minutes have been used. Since the current knowledge of the spectrum of the vertical fluxes of momentum and heat is in a pioneering stage, the optimum time-averaging period cannot be chosen with certainty; hence it was necessary to choose the time-averaging period arbitrarily. Subsequent analysis of results in this study will disclose that no important differences were found using 5- or 25-min averages.

The time-averaged data were analyzed with a digital computer and the least-squares technique for curve fitting. The analysis included the following steps: (1) determination of the roughness parameter for each site; (2) investigation of the vertical distribution of $z_0/L$; (3) determination of $\alpha'$; and (4) empirical investigation of other diabatic profiles.

RESULTS

Boundary characteristics. The roughness parameter $z_0$ is introduced in (2) as the constant of integration which is the theoretical finite height at which the wind speed vanishes. Its magnitude is interpreted as a function of the surface configuration upstream from the site. If one recognizes $z_0$ as a surface characteristic and not a stability parameter, it is logical to obtain its value for each site from the neutral profiles and, subsequently, to impose this lower boundary condition on all profiles regardless of stability. Since the wind profile must approach a neutral one near the surface during any stability regime, all cases were analyzed up to 2 and 4 m to determine $z_0$.

The roughness parameter was obtained by least-squares analysis of the data by use of the regression relation

$$ u = A \ln z + B $$

where $A = U_0/k$ and $B = -(U_0/k) \ln z_0$. A preferred $z_0$ was then obtained (from the numerous profiles) by least-squares analysis of the $A$ and $B$ values by use of the regression relation

$$ B = MA $$

where $z_0 = e^{-M}$. The dispersion of $M$, and subsequently of $z_0$, was estimated with the statistical parameter

$$ SE(M) = (\sigma_M/\sigma_M)(1 - r^2)/(N - 2)^{1/2} $$

where $SE(M)$ is the standard error of estimate of $M$.

The results are shown in Table 2. In general, $z_0$ at the sites could be explained as a function of seasonal variations in surface roughness and not as a function of wind speed and direction with but one exception. There is an oat patch west of station A which apparently created sufficient mechanical turbulence so that the observed $z_0$ was approximately double that obtained when the wind direction was other than 215 to 270° (21 cases). It is interesting to observe that an obstacle nearly 100 m southwest of the tower was 'felt' by the wind profile. One may conclude that $z_0$ cannot be interpreted as a function only of the surface elements directly beneath the anemometers. In fact, the roughness parameter may vary considerably owing to the influence of upstream topography. Fortunately, it was possible to attribute the large observed variation in $z_0$ in the majority of cases, to seasonal alterations in the surrounding terrain rather than to upstream barriers. Furthermore, the high correlations, $r_{z_0}$, during each season imply an acceptable degree of horizontal uniformity for each season at the sites. In preparing the summary we found that it was necessary to exclude cases for which the wind speed was very low (arbitrarily chosen as <300 cm/sec at 8 m) and for which the lapse rate was extremely stable or unstable. In the present analysis, profiles with low wind speed occasionally indicated a $z_0$ which was an order of magnitude higher (or lower) than the average. Also, the extreme diabatic cases deviated significantly from the neutral profile even at the lowest levels. Elimination of these special profiles allowed evaluation of the preferred $z_0$ for each site.

In interpreting these results, we see that the grass around A apparently was more lush and 'rougher' in summer than in winter, since $z_0$ varied from 3.0 cm in summer to 0.4 cm in winter. At B the seasonal variation was opposite that found at A; $z_0$ varied from 1.5 cm in sum-
TABLE 2. Roughness Parameter

<table>
<thead>
<tr>
<th>Station A</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N$</td>
<td>$z_0$, cm</td>
</tr>
<tr>
<td>July</td>
<td>133</td>
<td>2.94</td>
</tr>
<tr>
<td>Aug.</td>
<td>133</td>
<td>3.13</td>
</tr>
<tr>
<td>Sept.</td>
<td>91</td>
<td>1.86</td>
</tr>
<tr>
<td>Oct.-Jan.</td>
<td>571</td>
<td>0.69</td>
</tr>
<tr>
<td>Feb.</td>
<td>56</td>
<td>0.40</td>
</tr>
<tr>
<td>July-Aug. (SW wind)</td>
<td>21</td>
<td>4.87†</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Station B</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N$</td>
<td>$z_0$, cm</td>
</tr>
<tr>
<td>June</td>
<td>331</td>
<td>1.52</td>
</tr>
<tr>
<td>July</td>
<td>290</td>
<td>1.65</td>
</tr>
<tr>
<td>Feb.</td>
<td>86</td>
<td>2.97</td>
</tr>
</tbody>
</table>

- Each profile fitted to 4 m.
- † Values of $z_0$ used in subsequent analysis.

mer to 3.0 cm in winter. The short hay apparently produced more mechanical turbulence in winter than in summer. The station records give no details which would allow an a priori prediction of the seasonal variation.

Stability parameter. The MO hypothesis is that $z/L$ characterizes full dynamical similarity when $r$ and $H$ are constant with height. MO did not measure $L$, but a quantity $B'$, to which $L$ was assumed to be uniquely related. $B'$ is defined as:

$$B' = g[T(z_1) - T(z_3)]/T_0 u^2(z_0)$$  \(12\)

where $z_1$, $z_2$, $z_3$ are three selected levels. Similarly, $L'$ was used in this study since measurements of $H$ were not available.

It is pertinent to consider various methods to determine $z/L'$ from standard micrometeorological data. Since $U_r/k$ may be determined independently in the regression technique, only the velocity and temperature gradients are needed to specify $z/L'$. Possible techniques are (1) linear interpolation about a fixed reference level, (2) quadratic interpolation about a fixed reference level, and (3) linear regression over several levels.

In the final analysis, linear interpolation about the 8-m level was applied. Table 3 contains the statistical distribution from a study of the vertical distribution of $z/L'$ for all 25-min cases.

Values of $z/L'$ were determined by linear interpolation about all possible levels and grouped by per cent frequency for the range -0.5 to +0.5. Outside this range the number of occurrences was usually low and the actual occurrences are given. In this sample only those cases having low mean wind speed (<3 m/sec at 8 m) are excluded.

The increase of the range of $|z/L'|$ with height is evident from Table 3. Therefore it may be concluded that the majority of individual profiles would show an increase of $|z/L'|$ with height. The individual $|z/L'|$ profiles are not shown, but many of these showed the increase.

With a view toward applying the log-linear profile within a radius of convergence, values for the 8-m level were chosen as the typical values of $z/L'$ to be used later.

Linear interpolation was preferred over the other methods because it is practical and relatively simple. Since many investigators will not have data for several levels, they may have to resort to a simple method. Therefore, values of $z/L'$ were calculated for 8 m by the method of linear interpolation. It should be emphasized, however, that the choice of method to specify $z/L'$ is arbitrary. Since $z/L'$ varies with height, caution must be employed when comparing these results with other works.

Coefficient of linear term. If we assume that
$z(8 \text{ m})/L'$ can be determined with an acceptable degree of accuracy by using linear interpolation, the least-squares technique may be applied to the log-linear profile to determine $U_r/K$ and $\alpha'/L'$. If the latter coefficient, multiplied by 800 for convenience, is plotted against $z(8 \text{ m})/L'$ as in Figures 3 and 4, the slope of the resultant curve may be interpreted as $\alpha'$. An abrupt change in slope near the origin in each figure is apparent.

If straight lines were fitted to each regime in Figures 3 and 4, the value of $\alpha'$ for unstable regimes would vary from approximately 0.5 to 1.0 and for stable regimes, 4.0 to 6.0. This is a variation of almost an order of magnitude between lapse and inversion. Hence, separate least-squares fits of the data for both stability regimes were performed to determine the values of $\alpha'$; the results are shown in Tables 4 to 6. These results are the outcome of a few of the several experiments which were designed to determine the sensitivity of the values of $\alpha'$ to the analytical techniques. In these experiments all cases in which the temperature profile did not monotonically increase or decrease with height were omitted. Clearly, the similarity hypothesis and the technique to obtain $z/L'$ imply this assumption. Besides this precaution, a few cases which yielded an $\alpha'/L'$ coefficient of opposite sign to the independent $z/L'$ were also omitted. These latter are probably due to random errors during near-neutral stability. A pilot study showed that the omission of these values near the origin has little effect on the results.

The effect on $\alpha'$ of the arbitrary choice of the
Fig. 3. Determination of stability parameter versus regression coefficient for more than 350 twenty-five-minute cases to 16 m.

8-m level to determine $z/L'$ is shown in Table 4. Here, the least-squares $\alpha'$ is found for $z/L'$ interpolated about both 4 m and 8 m. The lapse

values of $\alpha'$ for each $z/L'$ were approximately the same; however, the inversion values of $\alpha'$ differed substantially as the stability increased.

In Table 5 the results of fitting the wind speed to the log-linear profile up to 16 m versus 32

TABLE 4. Effect of Stability Parameter on Monin and Obukov Parameter $\alpha'$ (25-min cases)*

<table>
<thead>
<tr>
<th>Range $z/L'$</th>
<th>N</th>
<th>Mean $z/L'$</th>
<th>Mean $\alpha'z/L'$</th>
<th>Implied $\alpha'$</th>
<th>N</th>
<th>Mean $z/L'$</th>
<th>Mean $\alpha'z/L'$</th>
<th>Implied $\alpha'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.0 to -0.8</td>
<td>2</td>
<td>-0.90</td>
<td>-0.68 ± 0.06</td>
<td>0.76</td>
<td>1</td>
<td>-0.66</td>
<td>-0.41</td>
<td>0.63</td>
</tr>
<tr>
<td>-0.8 to -0.6</td>
<td>2</td>
<td>-0.61</td>
<td>-0.46 ± 0.01</td>
<td>0.65</td>
<td>4</td>
<td>-0.50</td>
<td>-0.35 ± 0.03</td>
<td>0.71</td>
</tr>
<tr>
<td>-0.6 to -0.4</td>
<td>14</td>
<td>-0.51</td>
<td>-0.45 ± 0.13</td>
<td>0.87</td>
<td>19</td>
<td>-0.26</td>
<td>-0.22 ± 0.07</td>
<td>0.87</td>
</tr>
<tr>
<td>-0.4 to -0.2</td>
<td>41</td>
<td>-0.28</td>
<td>-0.33 ± 0.11</td>
<td>1.16</td>
<td>87</td>
<td>-0.09</td>
<td>-0.14 ± 0.07</td>
<td>1.51</td>
</tr>
<tr>
<td>-0.2 to 0.0</td>
<td>56</td>
<td>-0.13</td>
<td>-0.20 ± 0.17</td>
<td>1.60</td>
<td>174</td>
<td>0.10</td>
<td>0.55 ± 0.47</td>
<td>5.50</td>
</tr>
<tr>
<td>0.0 to 0.2</td>
<td>174</td>
<td>0.10</td>
<td>0.55 ± 0.47</td>
<td>5.50</td>
<td>174</td>
<td>0.10</td>
<td>0.55 ± 0.47</td>
<td>5.50</td>
</tr>
<tr>
<td>0.2 to 0.4</td>
<td>37</td>
<td>0.26</td>
<td>1.49 ± 0.48</td>
<td>5.71</td>
<td>15</td>
<td>0.29</td>
<td>0.95 ± 0.38</td>
<td>3.25</td>
</tr>
<tr>
<td>0.4 to 0.6</td>
<td>7</td>
<td>0.50</td>
<td>2.34 ± 0.65</td>
<td>4.64</td>
<td>2</td>
<td>0.46</td>
<td>1.30 ± 0.28</td>
<td>2.83</td>
</tr>
<tr>
<td>0.6 to 0.8</td>
<td>1</td>
<td>0.68</td>
<td>3.61</td>
<td>5.17</td>
<td>1</td>
<td>0.92</td>
<td>2.23</td>
<td>2.42</td>
</tr>
<tr>
<td>0.8 to 1.0</td>
<td>1</td>
<td>0.83</td>
<td>4.46</td>
<td>5.39</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Least-Squares $\alpha'$ $r$ Least-Squares $\alpha'$ $r$

Unstable 115 0.81 ± 0.01 0.99 111 0.68 ± 0.01 0.99
Stable 220 5.25 ± 0.005 0.99 197 2.99 ± 0.02 0.99

* Sample was all 25-min cases fitted to 16 m except for February B cases; cases with negative $\alpha'$ omitted.
† $z/L'$ determined by linear interpolation about indicated level. Values of implied $\alpha'$ obtained by dividing $\alpha'z/L'$ by $z/L'$.
‡ Tolerances are standard deviations of the series of $\alpha'z/L'$ values.
m are shown. In this experiment the \( \alpha' \) values were nearly equal during inversion conditions but varied during lapse conditions.

Table 6 illustrates the least-squares \( \alpha' \) for 2 months for the 5-min cases; similar results for the other months are not shown.

In Tables 4 to 6 the paired values of \( z/L' \) and \( \alpha'z/L' \) have been grouped in ten classes according to the value of the independent variable \( z/L' \); they were then averaged and recorded. The least-squares \( \alpha' \) for each regime was obtained by a least-squares analysis of the averaged values. The implied \( \alpha' \) values were obtained by dividing the mean \( \alpha'z/L' \) by the mean \( z/L' \) for that class. It is apparent from a cursory inspection of

**TABLE 6. Parameter \( \alpha' \) (5-min cases fitted to 32 m, station B)**

<table>
<thead>
<tr>
<th>Range ( z/L' )</th>
<th>July</th>
<th></th>
<th>February</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean ( z/L' )</td>
<td>Mean ( \alpha'z/L' )</td>
<td>Implied ( \alpha' )</td>
</tr>
<tr>
<td>-1.0 to -0.8</td>
<td>9</td>
<td>-0.90 ± 0.35</td>
<td>0.39</td>
</tr>
<tr>
<td>-0.8 to -0.6</td>
<td>9</td>
<td>-0.71 ± 0.33</td>
<td>0.46</td>
</tr>
<tr>
<td>-0.6 to -0.4</td>
<td>8</td>
<td>-0.46 ± 0.23</td>
<td>0.50</td>
</tr>
<tr>
<td>-0.4 to -0.2</td>
<td>21</td>
<td>-0.30 ± 0.18</td>
<td>0.62</td>
</tr>
<tr>
<td>-0.2 to 0.0</td>
<td>29</td>
<td>-0.15 ± 0.14</td>
<td>0.90</td>
</tr>
<tr>
<td>0.0 to 0.2</td>
<td>146</td>
<td>0.13 ± 0.24</td>
<td>7.33</td>
</tr>
<tr>
<td>0.2 to 0.4</td>
<td>71</td>
<td>0.27 ± 0.17</td>
<td>5.75</td>
</tr>
<tr>
<td>0.4 to 0.6</td>
<td>13</td>
<td>0.48 ± 0.47</td>
<td>4.95</td>
</tr>
<tr>
<td>0.6 to 0.8</td>
<td>2</td>
<td>0.65 ± 0.32</td>
<td>4.60</td>
</tr>
<tr>
<td>0.8 to 1.0</td>
<td>1</td>
<td>0.91 ± 0.42</td>
<td>4.70</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Least-Squares ( \alpha' )</th>
<th>( r )</th>
<th>Least-Squares ( \alpha' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unstable</td>
<td>76</td>
<td>0.44 ± 0.01</td>
</tr>
<tr>
<td>Stable</td>
<td>233</td>
<td>4.78 ± 0.02</td>
</tr>
</tbody>
</table>

* Tolerances are standard deviations of the series of \( \alpha'z/L' \) values.
Fig. 5 Root-mean-square residual (cm/sec) for log profile (upper) and log-linear profile (lower) for 454 twenty-five-minute cases as function of stability and height.
Tables 4 to 6 that $\alpha'$ is not a universal constant. It varies considerably with stability, from approximately 0.2 in extremely unstable regimes to 3.0 in slight lapse regimes and from 6 to 9 in slight inversion to 3 or 4 in extremely stable regimes. Most likely the fundamental cause of the variation was that $\alpha'$ is not a constant but a function of stability. It is still possible, however, that a constant value of $\alpha'$ in the two different stability regimes can be used for practical purposes. If one chooses $\alpha' \approx 0.6$ for lapse conditions and $\alpha' \approx 5.0$ for inversion conditions, two useful forms of the log-linear profile are obtained. When the stability is nearly neutral, $L'$ is large and the error in estimating the wind profile due to an error in $\alpha'$ is small. Very few wind profiles occurred during extreme conditions, $|z/L'| > 0.5$; therefore, the error in $\alpha'$ becomes important only for infrequent cases. Before these statements can be accepted it is necessary to test the two values of $\alpha'$ on independent data. This is not accomplished in the present work.

Several questions may now be asked:

(a) To what extent can the log-linear profile represent the actual measurements of $\bar{u}$?

(b) Can the coefficient of the quadratic term (say $\beta'$) in the Taylor expansion of $f(z/L')$ be adequately determined?

(c) Are there other empirical profiles which could be applied within the general theory?

Answers to these will aid the practical micro-meteorologist and, in turn, should provide additional information for the theorist.

Question (a) is partially answered in Figures 5 and 6. All 25-min cases, including those removed previously, were fitted to 32 m by means of least squares; both the log profile and the log-linear profile were used. Individual values of $\alpha'$ were used, not the two practical values suggested above. The 464 cases were separated into classes of $z/L'$ and the rms deviation was calculated for each level and class group. Figure 5 illustrates the expected result that the actual measurements deviated from the log profile as a function of both height and stability. On the other hand, when the log-linear profile was used (lower graph), the residual variance appeared to be reduced to that associated with inherent errors of measurement. The rms values averaged for all stability regimes are given in Table 7. At all levels, over 50% of the residual variance in the log profile was removed by the log-linear profile. For Figure 6 the 25-min cases were classified by wind speed at 8 m (1-m/sec intervals) before the rms values were calculated for each level and class. It is observed that the deviation from the log law was a function of wind speed and height. The deviation was largest at 32 m during light winds. As the wind speed increased, turbulent mixing probably tended to restore adiabatic conditions and the variance decreased. However, the log-linear profile adequately reduced the large deviation for all classes of wind speed and all levels.

In answer to question (b), since the log-linear profile reduces the residual variance to the data noise level, the coefficient $\beta'$ may not be found with any degree of certainty. Least-squares values of $\beta'$ for selected samples were found to be $0.001 \pm 0.004, 0.006 \pm 0.007$, etc. Similar results have been reported elsewhere (e.g., Taylor [1960]).

Considering question (c), one should recall that $S(z/L') \sim S(0) + \alpha' z/L'$. It is not unreasonable to write the diabatic profile in the empirical form,

$$\bar{u} = U_o k^{-1}[\ln(z/z_0) + Cz^\gamma] \quad (13)$$

where $C$ and $\gamma$ are constants to be determined and (13) will be called the log-power profile. It would be desirable that $\gamma = 1.0$, thereby confirming the log-linear profile. An experiment was
Texas A&M University researchers have previously found $\gamma \approx 0.5$ for 54 profiles from project Prairie Grass, O'Neill, Nebraska, in 1956 (W. H. Clayton, personal communication). In the present work, since the minimum in the standard error during lapse conditions was very shallow, the indicated $\gamma$ of 0.4 to 0.6 may not be significant. Therefore, it is suggested that the log-linear profile may be used for practical purposes during all conditions if one recognizes that $a'$ is a function of stability.

**Conclusions.** The MO hypothesis, as stated by the log-linear law, was confirmed for a broad range of stability only when $a'$ was interpreted to vary with stability. For practical estimation of the profile, it is suggested that $a'$ be assigned the MO value of 0.6 during lapse conditions and the value of 5.0 during inversion conditions. Hence, measurements of forecasts of the wind speed and temperature at two levels near 8 m and a knowledge of the roughness parameter, $z_o$, should be sufficient to estimate the mean wind speed to heights of 32 m.

Earlier, the Ellison profile (8) was introduced because it has received considerable theoretical attention. Since (8) may only imply a single value of $a'$, its usefulness is limited. It might be suggested that $a'$ be chosen to be constant and coefficients of higher-degree terms in the series expansion of $S$ be determined by least-square analysis. This will be difficult to accomplish since there is no criterion for choosing between the approximate values of $a'$ of 1 and 8 which occur at very slight lapse and inversion conditions, respectively.

It was shown that the roughness parameter can be determined to a practical degree of reproducibility even though the sites were not as uniform as the O'Neill, Nebraska, site. In addition, $z/L'$ was found to vary with height, as should be intuitively expected. This variation should not be interpreted as a deterrent to its practical application. It is well known that all previously suggested micrometeorological stability parameters also vary with height (e.g., Priestley [1959]). Since $L'$ did vary with height, the values of $a'$ found herein were dependent on the particular choice of $z/L'$, and some caution must be employed when the suggested $a'$ values are used by other investigators.

In the present analysis it has been necessary to introduce the inverse turbulent Prandtl num-

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**Fig. 6.** Root-mean-square residual (cm/sec) for log profile (upper) and log-linear profile (lower) for 464 twenty-five-minute cases as function of wind speed at 8 m and height.
Preferred ‘Y~

TABLE 8. Log-Power Profile, Mean Standard Error*

<table>
<thead>
<tr>
<th>Stability</th>
<th>N</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unstable</td>
<td>35</td>
<td>21.3</td>
<td>14.2</td>
<td>14.6</td>
<td>14.6</td>
<td>14.6</td>
<td>14.9</td>
<td>0.4</td>
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<tr>
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<td>17.4</td>
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<td>11.3</td>
<td>10.8</td>
<td>10.3</td>
<td>9.8</td>
<td>9.5</td>
</tr>
<tr>
<td>Aug.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unstable</td>
<td>45</td>
<td>19.3</td>
<td>13.1</td>
<td>13.4</td>
<td>13.4</td>
<td>13.5</td>
<td>13.7</td>
<td>13.9</td>
</tr>
<tr>
<td>Stable</td>
<td>165</td>
<td>15.5</td>
<td>10.7</td>
<td>10.6</td>
<td>10.2</td>
<td>9.8</td>
<td>9.4</td>
<td>9.2</td>
</tr>
<tr>
<td>Sept.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>100</td>
<td>15.3</td>
<td>7.8</td>
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<td>9.0</td>
<td>9.8</td>
<td>10.5</td>
<td>11.0</td>
</tr>
<tr>
<td>Stable</td>
<td>38</td>
<td>9.2</td>
<td>7.2</td>
<td>7.1</td>
<td>6.9</td>
<td>6.8</td>
<td>6.7</td>
<td>6.6</td>
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<tr>
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<td>8.5</td>
<td>8.6</td>
<td>8.8</td>
<td>9.1</td>
<td>9.4</td>
</tr>
<tr>
<td>Stable</td>
<td>354</td>
<td>8.7</td>
<td>8.2</td>
<td>8.7</td>
<td>8.1</td>
<td>7.6</td>
<td>7.3</td>
<td>6.9</td>
</tr>
<tr>
<td>Feb.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Unstable</td>
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<td>6.2</td>
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<td>8.8</td>
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<tr>
<td>Stable</td>
<td>42</td>
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<td>8.7</td>
<td>7.6</td>
<td>6.8</td>
<td>5.8</td>
<td>5.5</td>
<td>5.4</td>
</tr>
</tbody>
</table>

* Samples are all 5-min cases fitted to 16 m; data are arithmetic mean standard error for each regime.
† Minimum value of mean standard error determines preferred γ.

Variation of a' with stability is due to the variation of K\textsubscript{H}/K\textsubscript{K}. This question can be answered conclusively only when extensive measurements of the turbulent heat flux are available.

TABLE 9.* Effect of Stability on Log-Power Profile†

<table>
<thead>
<tr>
<th>Range</th>
<th>N</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>2</th>
<th>1.4</th>
<th>Preferred γ‡</th>
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</thead>
<tbody>
<tr>
<td>-1.0</td>
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<td>16.3</td>
<td>7.2</td>
<td>7.2</td>
<td>7.3</td>
<td>7.5</td>
<td>7.9</td>
<td>8.3</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>4</td>
<td>16.3</td>
<td>7.2</td>
<td>7.2</td>
<td>7.3</td>
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</tr>
<tr>
<td>0.5</td>
<td>33</td>
<td>15.5</td>
<td>8.4</td>
<td>8.6</td>
<td>9.0</td>
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<td></td>
</tr>
<tr>
<td>-0.4</td>
<td>54</td>
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<td>8.4</td>
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<td>9.1</td>
<td>9.5</td>
<td>10.0</td>
<td></td>
</tr>
<tr>
<td>-0.3</td>
<td>54</td>
<td>15.4</td>
<td>8.1</td>
<td>8.4</td>
<td>8.7</td>
<td>9.1</td>
<td>9.5</td>
<td>10.0</td>
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</tr>
<tr>
<td>-0.2</td>
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<td>9.8</td>
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<td>10.2</td>
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<tr>
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<td>9.7</td>
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<td>11.3</td>
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<td>10.0</td>
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<td>9.9</td>
<td>9.9</td>
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</tr>
<tr>
<td>0.2</td>
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<td>19.4</td>
<td>8.8</td>
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<td>7.3</td>
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<td>8.7</td>
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<td>9.0</td>
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<td>11.8</td>
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<td>16.9</td>
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</tr>
<tr>
<td>0.5</td>
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<td>34.9</td>
<td>8.9</td>
<td>9.0</td>
<td>10.3</td>
<td>12.1</td>
<td>13.9</td>
<td>15.5</td>
<td></td>
</tr>
</tbody>
</table>

* Entries are mean standard errors for N profiles for various values of γ.
† All 5-min cases from June station B were fitted to 16 m using log-power profile.
‡ Minimum value of mean standard error determines preferred γ.
Acknowledgments. I wish to express my appreciation to Professor Robert O. Reid for his guidance and encouragement throughout the preparation of this paper. Special thanks are given Dr. William Clayton for providing the data from the Dallas Tower Project.

The National Space and Aeronautics Administration provides the fellowship grant which supports my studies. The data were collected under Signal Corps contract DA 36-039 AMC-02195(E).

APPENDIX

\( \bar{u} \), mean wind speed for height \( z \).
\( z \), height above the surface.
\( k \), von Kármán's constant.
\( U_* \), friction velocity, defined by \( (\tau_0/\rho)^{1/3} \).
\( \tau_0 \), surface shearing stress.
\( \rho \), surface air density.
\( z_o \), roughness parameter.
\( S \), logarithmic wind shear.
\( \xi \), stability parameter.
\( L \), Obukhov scale length.
\( C_p \), specific heat of dry air at constant pressure.
\( \theta \), potential temperature.
\( g \), gravitational field strength.
\( H \), turbulent heat flux.
\( \tau \), momentum flux.
\( \theta_o \), surface potential temperature.
\( L' \), gradient length.
\( K_{H} \), eddy thermal diffusivity.
\( K_M \), eddy viscosity.
\( \alpha \), MO universal wind profile constant.
\( \beta' \), wind profile coefficients of quadratic term.
\( \alpha' \), log-linear profile coefficient.
\( \eta \), Ellison profile parameter.
\( \psi \), time interval.
\( A, B, M \), regression coefficients.
\( B' \), defined by equation 12.
\( \sigma \), standard deviation.
\( r \), product-moment correlation coefficient of \( A \) and \( B \).
\( N \), number of profiles in sample.
\( T \), mean air temperature for height \( z \).
\( \gamma, C \), log-power coefficients.

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