Steady Coastal Upwelling Induced by an Along-Shore Current

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(Manuscript received 14 December 1970, in revised form 8 March 1971)

Abstract

Studies of frictionally induced motions of continental shelf water near a large-scale ocean current reveal the possibility of strong coastal upwelling whenever the straight coast lies to the left of the current direction in the Northern Hemisphere and to the right of it in the Southern Hemisphere. In a homogeneous ocean with linear dynamics, this is apparently a consequence of the adjustment necessitated by the presence of bottom friction over the continental shelf which causes reduction of the Coriolis acceleration and allows the development of an onshore flow near the bottom due to the unbalanced pressure gradient. As a result, a one-sided subsurface mass convergence is established. In a vertical section normal to the coast, viewed in the direction of the current, the streamlines thus form a clockwise gyre in the Northern Hemisphere and a counterclockwise gyre in the Southern Hemisphere. In both cases, strong coastal upwelling appears. In the presence of a subsurface countercurrent offshore, the region of upwelling is displaced toward the shelf edge. Inshore sinking of coastal water occurs.

The analysis employs Fourier series expansions in distance away from the coast and depth from the sea surface. The final solution of the velocity field is expressed in terms of convergent infinite series. Numerical examples simulating major upwelling areas (Oregon, Peru and Gulf of Mexico Coast) are then worked out and discussed to illustrate the essential dynamics.

1. Introduction

Persistent upward motion of coastal water (or, simply, coastal upwelling) along their eastern boundary has long been noted as a basic feature of the general circulation of the major oceans (Wooster and Reid, 1961; Smith, 1968). It is also unique in its location, for coastal upwelling is almost exclusively an eastern boundary phenomenon. This is usually attributed to the combination of the local atmospheric circulation and the presence of adjacent continental boundaries. The major subtropical anticyclones which cover separately the Northern and Southern Hemisphere portions of each ocean characteristically create equatorward winds along the eastern boundary. As a direct response of the ocean, an offshore mass transport (and thus a one-sided divergence) in the first hundred meters or so is created. The result is upwelling of subsurface waters along the coast. Although such an explanation appears quite reasonable, it should be noted that from the point of view of the total coastal circulation, the upwelling of any coastal water is really not influenced by surface wind alone. The interrelation between coastal upwelling and background circulations induced by large-scale offshore currents must be examined before a definitive analysis of coastal upwelling can be achieved. The possibility of bottom water upwelling along a continental slope, for example, has been noted by Cochrane (1969). This paper investigates these interrelationships.

In order to elucidate the essential dynamics involved with the least amount of mathematical complication, a simple linear model of a homogeneous ocean with a straight vertical coast and a flat continental shelf is employed. At the seaward edge of the shelf region, the horizontal velocity of the offshore current is prescribed as a function of depth. There is no wind stress at the free surface. The shelf circulation is thus ideally conceived as being induced by the offshore current.

Insofar as the problem remains linear, the no-flux condition at the free surface can be relaxed in principle with no difficulty. Solutions including surface wind stress will be discussed at length elsewhere.

In reality, of course, the ocean is also stratified. Consequently, any vertical motion in a real ocean must be compatible with the balance of buoyancy flux. This limits the magnitude of the actual vertical velocity as compared with that computed here. However, it is reasonable to expect that the essential kinematic structure in the real ocean would still be reflected to a large extent in a homogeneous theory.

2. Formulation

Consider a straight vertical coast in the $y^*$ direction. The continental shelf, or coastal plain, extends seaward in the $x^*$ direction to a distance $b^*$, at which point a
steady current is prescribed, having \( z^* \) and \( y^* \) components, respectively,

\[
\begin{align*}
\bar{u}^*(b^*,z^*) &= \sum_{m=0}^{\infty} K_m^* \cos \left( \frac{2m+1}{2d^*} \pi z^* \right), \\
\bar{v}^*(b^*,z^*) &= \sum_{m=0}^{\infty} F_m^* \cos \left( \frac{2m+1}{2d^*} \pi z^* \right),
\end{align*}
\]

where \( z^* \) is the vertical coordinate positive upward from the equilibrium sea level. A schematic of this configuration is presented in Fig. 1.

The above velocity components clearly satisfy the no-slip condition at the bottom, \( z^* = -d^* \). At the free surface, they also satisfy the no-flux condition, provided that term-by-term differentiation is allowed. This is nearly always the case, as in practice only a finite number of Fourier components are necessary to represent any reasonable velocity profile.

It should be noted that (1) and (2) do not contain the \( y^* \) coordinate. On an \( f \) plane, the near-shore circulation is assumed independent of \( y^* \).

Upon neglecting the nonlinear acceleration terms and assuming constant density and hydrostatic equilibrium, the nondimensional governing equations for steady-state conditions can readily be written as

\[
\frac{\partial \bar{r}}{\partial x^*} + \frac{\partial \bar{w}}{\partial z^*} = 0,
\]

where \( \bar{W} = u + iv \), and \( \bar{r} \) represents the nondimensional sea surface displacement. The double sign convention adopted is that the upper (lower) sign applies in the Northern (Southern) Hemisphere. Here, \(\bar{w}\) is the nondimensional vertical velocity component and is proportional to the dimensional vertical velocity. The constant of proportionality includes \( C \), a typical horizontal speed, and \( A_V \) and \( A_H \), the constant vertical and horizontal eddy viscosities. The speed \( C \) is naturally chosen as the maximum forcing velocity at the continental shelf break at the open ocean side of the shelf.

Mathematically,

\[
\bar{w}^* = (A_V/A_H)C \bar{w}.
\]

The horizontal velocities are also nondimensionalized with \( C \), i.e.

\[
\begin{align*}
\left[ \begin{array}{c} \bar{x}^* \\ \bar{y}^* \\ \bar{z}^* \\ \bar{v}^* \\ \bar{u}^* \end{array} \right] &= C \left[ \begin{array}{c} x \\ y \\ z \\ v \\ u \end{array} \right] .
\end{align*}
\]

Using \( g \) as gravity and \( \bar{r}^* \) as the free surface displacement, the additional nondimensionalization is

\[
\begin{align*}
\bar{r}^* &= \frac{(A_H/f)^2}{g}, \\
\bar{x}^* &= (A_H/f)x, \\
\bar{y}^* &= (A_H/f)y, \\
\bar{z}^* &= (A_H/f)z.
\end{align*}
\]

As a result, the asterisks that appear in (1) and (2) are now dropped, provided one defines

\[
\begin{align*}
b &= (f/A_H)^2b^*, \\
d &= (f/A_V)^2d^*.
\end{align*}
\]

It should be mentioned, in terms of the present scaling, that the nonlinear acceleration terms that are being omitted in the formulation are of the order of the Rossby number

\[
\text{Ro} = C/(fA_H)^2.
\]

In the ocean if we assume the horizontal eddy viscosity \( A_H \) has a typical value of \( 10^7 \text{ cm}^2 \text{ sec}^{-1} \), then \( C \approx 10 \text{ cm sec}^{-1} \) and \( \text{Ro} \approx O(10^{-4}) \). However, in strong currents such as near a western boundary, \( \text{Ro} \) may be of order unity and the present theory does not apply.

The physical dimensional problem contains six parameters: \( b^*, d^*, f, C, A_V, A_H \). The strength of the nondimensional problem is that it contains only two parameters, \( b \) and \( d \). The shape of the offshore velocity is explicit in both cases. A particular solution of the present theory will apply if the geometric parameters, \( b^*, d^*, f \) imply eddy viscosities which together with \( C \) yield a small Rossby number.

3. Solutions

To solve (3) and (4), it is convenient to introduce the Fourier series representation for the complex velocity \( \bar{W} \) and the pure number unity in the \( z \) dimension as

\[
\bar{W} = \sum_{m=-\infty}^{\infty} W_m(x) \cos \left( \frac{2m+1}{2d^*} \pi x \right),
\]

\[
1 = \sum_{m=-\infty}^{\infty} (\frac{2m+1}{2d^*} \pi x)^m.
\]

Here, \( W_m(x) \) is the Fourier coefficient of the complex velocity \( \bar{W} \) at \( x \). The solution to the governing equations is then

\[
\text{The solutions in the Southern Hemisphere are obtained from the}
\]

\[
\text{Northern Hemisphere by replacing the upper (lower) sign in}
\]

\[
\text{equations and by changing the signs of} \ W_m(x).
\]

Fig. 1. A schematic of the axes of reference for coastal upwelling computation.
In terms of the Fourier coefficients, (3) is reduced to an ordinary differential equation in \( x \) for each \( m \), i.e.,
\[
W_{m}'' - \left[ \left( \frac{2m+1}{2d} \right)^{2} - \frac{1}{\pi} \right] W_{m}'' = -\frac{4}{\pi} \frac{(-1)^{m+1}}{2m+1},
\]
where the primes denote the order of differentiation.

Let the free surface slope \( \xi' \) be expanded in a Fourier series in the \( x \) domain with coefficients \( C_{s} \) yet to be determined; thus, we have
\[
\xi' = \sum_{s=0}^{\infty} C_{s} \cos \left[ \frac{\pi(2s+1)(b+x)}{2b} \right].
\]

On the basis of this expansion, the particular solution \( W_{m}^{p} \) follows immediately in terms of \( C_{s} \) as
\[
W_{m}^{p} = \sum_{s=0}^{\infty} (a_{m} + ib_{m})C_{s} \cos \left[ \frac{\pi(2s+1)(b+x)}{2b} \right],
\]
where
\[
a_{m} = \frac{4 \frac{(-1)^{m+1}}{\pi 2m+1}}{1 + \left[ \left( \frac{2m+1}{2b} \right)^{2} + \left( \frac{2m+1}{2d} \right)^{2} \right]^{1/2}},
\]
\[
b_{m} = \frac{-4 \frac{(-1)^{m+1}}{\pi 2m+1}}{1 + \left[ \left( \frac{2m+1}{2b} \right)^{2} + \left( \frac{2m+1}{2d} \right)^{2} \right]^{1/2}}.
\]

The homogeneous solution \( W_{m}^{H} \) of \( W_{m} \), on the other hand, satisfies (14) in absence of any free surface slope. Note that the complex function \( W_{m}^{H} \) as given by (16) vanishes at the coast \( (x=0) \). At the seaward edge of the coastal strip,
\[
W_{m}^{H}(b) = -\sum_{s=0}^{\infty} (a_{m} + ib_{m})C_{s}.
\]

Consequently, the complex function \( W_{m}^{H} \) must satisfy the boundary conditions
\[
W_{m}^{H}(0) = 0,
\]
\[
W_{m}^{H}(b) = K_{m} + \sum_{s=0}^{\infty} a_{m}C_{s} + i(F_{m} + \sum_{s=0}^{\infty} b_{m}C_{s}).
\]

As a result, the homogeneous solution for each \( m \) becomes
\[
W_{m}^{H}(x) = \left[ \pm (F_{m} + \sum_{s=0}^{\infty} b_{m}C_{s})e_{m} + (K_{m} + \sum_{s=0}^{\infty} a_{m}C_{s})d_{m} \right] \sinh(p_{m}x) \cos(q_{m}x)
\]
\[
+ i\left[ (F_{m} + \sum_{s=0}^{\infty} b_{m}C_{s})d_{m} - (K_{m} + \sum_{s=0}^{\infty} a_{m}C_{s})e_{m} \right] \cosh(p_{m}x) \sin(q_{m}x)
\]
\[
+ i\left[ (F_{m} + \sum_{s=0}^{\infty} b_{m}C_{s})e_{m} \pm (K_{m} + \sum_{s=0}^{\infty} a_{m}C_{s})d_{m} \right] \cosh(p_{m}x) \sin(q_{m}x),
\]
where
\[
p_{m} = 2 \left( \left( \frac{2m+1}{2d} \right)^{2} + 1 \right) - \left( \frac{2m+1}{2d} \right)^{2},
\]
\[
q_{m} = \frac{1}{2p_{m}},
\]
\[
e_{m} = \frac{2 \cosh(p_{m}b) \sin(q_{m}b)}{\cosh(2p_{m}b) - \cos(2q_{m}b)},
\]
\[
c_{m} = \frac{2 \sinh(p_{m}b) \cos(q_{m}b)}{\cosh(2p_{m}b) - \cos(2q_{m}b)},
\]
\[
d_{m} = \frac{2 \sinh(p_{m}b) \cos(q_{m}b)}{\cosh(2p_{m}b) - \cos(2q_{m}b)}.
\]

The sum of the particular solution (16) and the homogeneous solution (22) constitutes the most general solution to (14) for each \( m \). The complete solution to the coastal circulation model is thus given collectively by all the Fourier modes in the \( z \) domain as
\[
W = \sum_{m=0}^{\infty} W_{m}(x; C_{m}) \cos \left( \frac{2m+1}{2d} \pi z \right).
\]
transport in the direction normal to the coast, namely

$$\int_{-d}^{t} \dot{u} ds = 0, \quad \text{for all } x.$$  \hspace{1cm} (28)

Since the nondimensional sea-surface displacement is small compared to the total depth $d$, the upper limit $t$ is replaced by zero in subsequent computation. This implies that the dynamic effect of the pressure gradient is explicitly included in (3) but its small role in determining the mass balance is neglected.

Upon substituting for $u$ the real part of (27), the algebraic equation for $C_s$ is

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{2m+1} \bar{I}_m C_s + \sum_{m=0}^{\infty} \frac{(-1)^m}{2m+1} (\pm \epsilon_m I_m + d J_m) \sum_{n=0}^{\infty} \bar{b}_n C_n$$

$$+ \sum_{m=0}^{\infty} \frac{(-1)^m}{2m+1} (\pm \epsilon_m J_m + d I_m) \sum_{n=0}^{\infty} \bar{a}_n C_n$$

$$+ \sum_{m=0}^{\infty} \frac{(-1)^m}{2m+1} (d I_m + \epsilon_m J_m) \sum_{n=0}^{\infty} \bar{a}_n C_n$$

$$= 0. \hspace{1cm} (29)$$

In the above, $\bar{I}_m$ and $\bar{J}_m$ are, respectively, the Fourier cosine coefficients of $\sinh(\rho_m x) \cos(q_m z)$ and $\cosh(\rho_m x) \sin(q_m z)$. Precisely, the following relations hold:

$$\sinh(\rho_m x) \cos(q_m z) = \sum_{x=0}^{\infty} I_m \cos \left[ \frac{\pi(2x+1)(b+x)}{2b} \right], \hspace{1cm} (30)$$

$$\cosh(\rho_m x) \sin(q_m z) = \sum_{x=0}^{\infty} J_m \cos \left[ \frac{\pi(2x+1)(b+x)}{2b} \right]. \hspace{1cm} (31)$$

When truncated in $s$, Eq. (29) can be solved on a digital computer for $C_s$ by a standard Gaussian elimination subroutine. Successive truncations at $s=1(1)40$ are employed. The solution converges quickly and $C_s$ drops in magnitude rapidly with increasing $s$; an example will demonstrate this.

4. Examples of induced upwelling

Once the $C_s$'s are known, the function $W_m$ can be computed immediately. The imaginary part of (27) yields isotachs for $v$ in the $x,z$ plane while the real part of (27) together with the equation of continuity (4) yields $u$ and $w$, the upwelling velocity.

It is computationally convenient to introduce a streamfunction $\psi$ in the plane normal to the coast such that

$$u = \frac{\partial \psi}{\partial z}, \quad w = -\frac{\partial \psi}{\partial x}. \hspace{1cm} (32)$$

In terms of $W_m$, this leads to

$$\psi(x,z) = \sum_{m=0}^{\infty} \frac{2d}{(2m+1)x} \times \text{Re} \left[ W_m(\frac{2m+1}{2d}) \right] \sin \left( \frac{2m+1}{2d} \pi s \right).$$

Computations of $v$ and $\psi$ are made for four ideal coastal circulation cases, each of which corresponds roughly to an important near-shore region under the influence of offshore currents in the real ocean. The vertical and horizontal eddy viscosity may be taken, respectively, to be $10^4$ and $10^5 \text{ cm}^2 \text{ sec}^{-1}$ if the reader wishes to estimate dimensional velocities from the following ideal cases. These values of eddy viscosities are not critical to the details of the solution.

a. Case 1. Oregon coast

The values of the appropriate parameters used are:

$$F_0=1, \quad F_n=0 \quad \text{for all } m \neq 0; \quad K_0=0.03, \quad K_1=0.1, \quad K_n=0 \quad \text{for all other } m \text{'s}; \quad b=4, \quad d=16.$$  

This case roughly corresponds to the continental shelf region off the coast of Oregon. The model geometry implies a dimensional depth of the order of 160 m and a dimensional width of 40 km. The offshore velocity profile is composed of an along-shore component which varies as a quarter of a cosine wave in $z$ and a small flow normal to the coast that satisfies the constraint (28). This forcing velocity is constructed on the basis of actual current measurements (Collins and Pattullo, 1970).

The results of computation are presented in Figs. 2 and 3 which show, respectively, the $v$ isotachs and the streamfunction contours in the $x,z$ plane. The total flow pattern reflects strongly the influence of friction.

Fig. 2. The computed isotachs in a cross section normal to the Oregon coast, in units of $10^4$. The abscissa and the ordinate coincide, respectively, with the bottom and the coast. All values are nondimensional.
The along-shore velocity $v$ over the shelf decreases gradually toward the coast where it vanishes. Near the bottom where the vertical friction becomes important, the along-shore velocity is again reduced, which reduces the Coriolis acceleration. Therefore, there is an excess of the pressure force toward the shore. As a result, an onshore bottom current is established. As there can be no net mass flux in the $x$ direction, coastal upwelling occurs and together with it, an offshore return flow forms in the upper layer. Part of the return flow is recirculated and causes sinking motion at the outer edge of the continental shelf. The rest escapes to the open sea in accordance with the prescribed boundary condition. If $u(b, z) = 0$, all the return flow sinks.

As is easily seen in Fig. 3, the area of upwelling extends seaward to an offshore distance of nearly three-quarters of $b$. The maximum upwelling takes place at $x = 2$ and $z = -8$ where $w^* = 3.5 \times 10^{-4}$ cm sec$^{-1}$, which is comparable with the commonly acknowledged magnitude for wind-induced upwelling (Smith, 1968).

The downward velocity at the shelf edge is unrealistically large. Stronger $u(b, x)$ will, however, reduce the recirculation and thus the offshore sinking. In fact, an alternative boundary condition which would eliminate or at least minimize the recirculation would be to require $\partial u/\partial x = 0$ at the shelf edge rather than specifying $u$ directly. The gradient condition is a less stringent one, but does lead to some serious mathematical difficulties.

The sea-surface slope in this case is given by

$$i' = 0.52 \cos \left( \frac{\pi (b+x)}{2b} \right) + 0.07 \cos \left( \frac{3\pi (b+x)}{2b} \right) + \cdots,$$

which illustrates the rapid decrease of $C_i$. The subsequent coefficients decrease rapidly.

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**b. Case 2. Peru coast**

The values of the parameters used are: $F_1 = -1$, $F_m = 0$ for all $m \neq 1$; $K_m = 0$ for all $m$; $d = 10$, $b = 10$.

In the Southern Hemisphere this resembles the configuration off the Peru coast along which the Peru coastal current flows equatorward. Observations in this area have indicated the existence of an undercurrent which transports mass poleward (Sverdrup et al., 1942; Wooster and Gilmartin, 1961). For this reason, nonzero $F_1$ is chosen to impose a relatively strong countercurrent to the coastal circulation.

The sea surface slope in this case is given by

$$i' = -0.12 \cos \left( \frac{\pi (b+x)}{2b} \right) - 0.03 \cos \left( \frac{3\pi (b+x)}{2b} \right) - 0.01 \cos \left( \frac{5\pi (b+x)}{2b} \right) - 0.009 \cos \left( \frac{7\pi (b+x)}{2b} \right) - 0.007 \cos \left( \frac{9\pi (b+x)}{2b} \right).$$

The computed circulation is presented in Figs. 4 and 5 in terms of the $v$ isotachs and streamlines, respectively. There is no imposed $u$ velocity in this case.

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**Fig. 3.** The computed streamlines in a cross section normal to the Oregon coast, in units of $10^3$. The arrowheads indicate the direction of flow. The abscissa and ordinate coincide, respectively, with the bottom and the coast. All values are nondimensional.

**Fig. 4.** The computed isotachs in a cross section normal to the Peru coast (units of $10^3$) with a strong poleward undercurrent. The abscissa and the ordinate coincide, respectively, with the bottom and the coast. All values are nondimensional.

**Fig. 5.** The computed streamlines ($10^3$) in a cross section normal to the Peru coast with a strong poleward undercurrent. The abscissa and the ordinate coincide, respectively, with the bottom and the coast. The arrowheads indicate the direction of flow. All values are nondimensional.
simulation of the Peru coastal water except in a small corner region where the equatorward surface flow dominates. Close to the coast and near the bottom, ageostrophic transport offshore is established, which results in subsidence of the water nearest the coast. However, at the shelf edge, the gradient of horizontal shear overrides the offshore transport due to the vertical shear gradient and produces onshore ageostrophic flow. Mathematically, since $|v_{ee}| > |u|$, $u$ yields onshore flow. Consequently, upwelling must take place in the center of the shelf region at some distance away from the coast.

For a weaker undercurrent, let $F_0 = -1/3$ and $F_1 = -2/3$. Then the zero isotach curves downward and intersects the solid boundary instead and coastal subsidence is weakened (Figs. 6 and 7).

In the absence of a poleward countercurrent, however, the streamlines in the $x,z$ plane as driven by friction can be expected to form only a single counterclockwise gyre.

c. Case 3. Gulf of Mexico coast

The values of the parameters used are: $F_0 = 1$, $F_1 = 0$ for all $m < 0$; $K_0 = 0.03$, $K_1 = 0.1$ and $K_\infty = 0$ for all other $m$; $b = 5$, $d = 10$.

In the Northern Hemisphere, this corresponds roughly to the situation off the Gulf of Mexico coast in Florida. The forcing function is identical with that in case 1. The dimensional depth and width of the model coastal strip are, respectively, 100 m and 50 km.

As in most parts in the east Gulf, the west Florida coastal water is strongly influenced by the Loop
current, which comes quite close to the edge of the shelf in the wintertime (Leipper, 1970). On the continental shelf inshore of the Loop current, regions of substantial sustained productivity have been found (notably the Florida middle ground). It is conjectured that these are sustained by upwelling induced by the Loop current.

The circulation pattern obtained on the basis of a steady current off the shelf is presented in Figs. 8 and 9. It resembles that found in case 1. A maximum upwelling velocity of $1.9 \times 10^{-4}$ cm sec$^{-1}$ is located $\approx 33$ km from the coast. Particularly important here is the onshore velocity at the shelf edge which allows deep water over the continental slope to rise carrying the high concentration of nutrients which enhances growth of marine life over the continental shelf.

The sea surface slope in this case is given by

$$f' = 0.5 \cos \left( \frac{\pi(b+x)}{2b} \right) + 0.08 \cos \left[ \frac{3\pi(b+x)}{2b} \right]$$

5. Discussion

In the above, the possibility of coastal upwelling induced by an offshore current is explored. The problem is approached from the standpoint of the complete near-shore circulation in the absence of wind stress. The results appear to indicate that locally the proposed mechanism is perhaps as important as the local atmospheric circulation. Along the eastern boundary of the major oceans, the combination of surface wind, coastline and offshore current tends to create favorable conditions for coastal upwelling, although the detailed analysis of the collective effect of these remains to be analyzed. Along the western boundary, inshore from the inertial currents, upwelling induced by the offshore current is still a possibility. But here the large value of the offshore current renders the theoretical result relatively tentative since the Rossby number may be large. The dynamics of the near-shore circulation (and thus coastal upwelling) on the western boundary is expected to be essentially nonlinear.

It should also be noted in spite of the presence of friction that the model appears to allow nonzero vertical velocities at the coast and at the seaward edge. In reality, of course, vertical shear layers must be present through which the flow adjusts in order to meet the appropriate side boundary conditions (Barcilon and Pedlosky, 1967). Fortunately, these boundary layers are usually very thin [O(100 m)] for the given ratio $A_w/A_H$. Consequently, the interior of the coastal region is free from dynamic interference of the frictional adjustments necessary on the sides. Detailed study of the influence of such near-shore boundary layers is, of course, urgently needed and is at present underway.

Acknowledgments. The work reported here is supported by the Office of Naval Research under Contract NONR-000014-67-A-0235(XX) with Florida State University. Mr. Bryan Travis has been mainly responsible for the numerical computations on the CDC 6400 at FSU. Additional calculations on the problem were programmed by Mr. James Smith at the Computing Facility of the National Center for Atmospheric Research, Boulder, Colo. NCAR is supported by the National Science Foundation.

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