Equatorial Jet in the Indian Ocean: Theory

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Abstract. A nonlinear numerical model and a simple analytical theory explain the basic features of the equatorial surface jet in the Indian Ocean recently reported by Wyrtki. The observed width of this transient current, 500 kilometers, is given theoretically by twice the baroclinic equatorial radius of deformation. The numerical model reproduces all Wyrtki's observations of this natural phenomenon.

During the transition periods between the two monsoon seasons, a narrow, high-speed surface jet flows along the equator from west to east across the entire Indian Ocean (1). We have been developing numerical models of the eastern boundary layer (2, 3) to study coastal upwelling. Wyrtki's report (1) encouraged us to use our model to explain the physics of this transient, wind-driven jet. In this report we derive theoretical estimates of the observed space and time scales given by Wyrtki.

In the numerical model the nonlinear, time-dependent primitive equations are solved for a two-layer, flat-bottomed ocean on a beta plane (4). The independent variables are $x$ (distance eastward), $y$ (distance northward), and $t$ (time). The dependent variables are the eastward and northward horizontal velocity components, $u$ and $v$, respectively, and the thickness, $h$, of each layer. The equations are

$$\frac{\partial v_1}{\partial t} + v_1 \cdot \nabla v_1 + k \times f v_1 = -g \nabla (h_1 + h_2 + D) \rho (v^* - v^1) + A \nabla v_1$$

$$\frac{\partial h_1}{\partial t} + \nabla \cdot (h_1 v_1) = 0$$

$$\frac{\partial v_2}{\partial t} + v_2 \cdot \nabla v_2 + k \times f v_2 = -g \nabla (h_1 + h_2 + D) \rho \left( v^* - v^1 \right) + A \nabla v_2$$

$$\frac{\partial h_2}{\partial t} + \nabla \cdot (h_2 v_2) = 0$$

$$v_1 = [u_1, v_1]$$

where $f$ is the Coriolis parameter; $g$ is the acceleration of gravity; $D$ is the height of the bottom topography above a reference level; $v^*$, $v^1$, and $v^2$ are the wind, interfacial, and bottom stresses; $\rho$ is the density of the ocean; $\Delta \rho$ is the density difference between the layers; and $A$ is the horizontal eddy viscosity.

These equations are well known, as are the details of their derivation (2). In this report we discuss a numerical solution of these equations and a simple analysis of a "stripped-down" linear version of this model.

Before presenting numerical results, we consider the simplest model of equatorial surface currents for the middle of the Indian Ocean, away from boundaries. If we let $u = u_1 - u_2$, $v = v_1 - v_2$, and $h = h_1$, Yoshida (5) has shown that the equations

$$\frac{\partial u}{\partial t} = -g \rho (h_1 + h_2 + D) \frac{\partial h}{\partial y}$$

$$\beta y = -\left( \frac{\partial \rho}{\partial \rho} \right) \frac{\partial h}{\partial y}$$

$$h \left( \frac{1}{H_1} + \frac{1}{H_2} \right) \frac{\partial h}{\partial t} = -\frac{\partial u}{\partial y}$$

(2)

may be used to deduce some understanding of the physics (6). In Eqs. 2, $\beta$ is the coefficient of the latitude variation of $f$, $v^*_x$ is the $x$ component of $v^*$, and $H$ is the initial value of $h$ (capital letters denote initial values).

When $v^*_x$ is a constant, impulsively applied, the north-south velocity, $v$, is a solution of the ordinary differential equation

$$L^* \frac{\partial v}{\partial y} - v = a L y$$

(3)
The fundamental scale length, \( L \), is known as the baroclinic equatorial radius of deformation. Time-dependent, baroclinic ocean motions always depend on the deformation radius. Here

\[
L = (C/\beta)^{1/2}
\]

\[
C = \left[ \frac{g \Delta \rho H}{\rho (H_1 + H_2)} \right]^{1/2}
\]

\[
a = \tau_e/\beta \rho H_1
\]

where \( C \) is the internal gravity wave speed. We expect a priori that the dominant baroclinic motions will be confined to within a distance \( L \) of the equator in the ocean interior. The solution to Eqs. 2 is illustrated in Fig. 1.

The measured values (or estimates) of these constants are \( \beta = 2.25 \times 10^{-18} \text{ cm}^{-1} \text{ sec}^{-1} \), \( \Delta \rho/\rho = 0.003 \), \( H_1 = 10^4 \text{ cm} \), and \( H_2 > 4H_1 \); thus, \( L = 277 \text{ km} \). Since the equatorial jet is symmetric about the equator, the width, \( 2L \), agrees quite well with Wyrtki's estimate of 500 km. The width of the jet is independent of the shape and strength of the wind field.

Equation 3 implies that far from the equator

\[
v = -\frac{\tau_e}{\beta \rho H_1}
\]

which is the classical Ekman drift solution. Near the equator, \( v \) approaches zero. Thus, from Eqs. 2, the eastward jet is driven directly by the wind stress. If the wind acts for 20 days at 0.5 dyne cm\(^{-2}\), we can expect the surface velocity to approach 100 cm sec\(^{-1}\). These estimates from the linear theory agree with the observations (1). After approximately 10 days, the linear model is invalid since substantial east-west pressure gradients develop. The numerical model does not have this deficiency.

The complete nonlinear numerical model was solved for a 5000-km-wide basin with rigid walls on the east and west and open north-south boundaries (3). The model was driven from rest by a uniform west wind. The wind stress tended to 0.5 dyne cm\(^{-2}\) with a time constant of 2 days. The ocean was initially at rest; \( h_1 = 120 \text{ m} \), \( h_2 = 480 \text{ m} \), and \( A = 10^6 \text{ cm}^2 \text{ sec}^{-1} \). The numerical techniques are described elsewhere (2, 3).

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**Fig. 1.** Solution to the linear model in non-dimensional units. The eastward jet, \( u/a \), and the pycnocline anomaly, \( (h - H)/H_1 \), are plotted at time \( (\beta L)^{-1} \). The jet flow decays to zero near \( 2L \); a weak westward flow is calculated from \( 2.3L \) and northward.

**Fig. 2.** Numerical solution at day 30. (a) Upper-layer transport vectors \( (h,v) \). The Ekman transport enters the equatorial region and then flows eastward in a strong equatorial jet. Along the eastern boundary the current flows north and south away from the equator in a narrow coastal jet. (b) Distribution of \( u \); the maximum eastward flow is 45 cm sec\(^{-1}\). (c) Rise (fall) of the thermocline from 120 m. This interface has risen 15 m in the west and fallen over 50 m along the eastern boundary. (d) Height (in centimeters) of the free surface above mean water level. There is a tilt of 20 cm across the basin.

**Fig. 3.** (a) Upper-layer transport after 60 days. A westward equatorial flow exists in the eastern ocean. The eastward-flowing jet leaves the equator at 3000 km and flows north and south. (b) East-west velocity, \( u \). The jet maximum (50 cm sec\(^{-1}\)) is near the western boundary.
The model has been integrated by using various wind patterns for over 75 days. In this short report we cannot properly present the model output. Instead, we choose to present distributions of $u_1(x,y)$ at 30 and 60 days, $h_2 - H_2$ (the pycnocline anomaly) at 30 days, and the free surface anomaly at 30 days. The upper layer transports are shown for 30 and 60 days.

After 30 days a narrow jet is well developed over the entire equatorial ocean (Fig. 2). The numerical grid was chosen such that all boundary currents are well resolved in the computations. This was accomplished by using a variable resolution grid (3). The equatorial jet reached a speed of 45 cm sec$^{-1}$ after 30 days. The tilt of the ocean surface is 20 cm across the basin, as observed by Wyrtki (1). The flow has induced upwelling in the western ocean and downwelling near the eastern boundary.

After 2 months the jet has reached a strength of 50 cm sec$^{-1}$ (Fig. 3) but the maximum has migrated far westward. The upper-layer flow shows a very interesting pattern. At the equator in the eastern third of the basin the pressure gradient is balanced by the wind stress. The surface current has reversed and flows westward. The equatorial jet separates from the equator at 3000 km and flows north and south in two narrow, strong (> 10 cm sec$^{-1}$) currents. Wyrtki does not report this current.

The thermocline (pycnocline) anomaly calculated after 60 days indicates a 25-m rise in the western ocean and a 45-m drop in the eastern ocean. These are very close to the rise and fall of the 20°C isotherm observed by Wyrtki. The tilt of the ocean surface is not shown after 60 days since the mean east-west tilt remains about 20 cm.

We have not shown the flow structure in the lower layer. It is interesting to note that the depth-averaged velocities are small, no more than 3 cm sec$^{-1}$. This implies that the currents seen in Figs. 2 and 3 are reversed in direction in the lower layer.

Each of Wyrtki's observations has been simulated in the numerical model. A more detailed report and interpretation of this simulation will be reported elsewhere (7).

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References and Notes

3. The numerical model developed by H. E. Hurlburt [thesis, Florida State University (1974)] was used for these computations. The details of the open boundary conditions, the variable mesh, numerical scheme, and so forth, are described in Hurlburt's thesis.
4. The Coriolis parameter, $f$, which depends on the sine of the latitude, is approximated by a linear function, $f = f_0 + f_2\lambda$, where $f_0$ is a reference value (here 0) and Cartesian coordinates are used.
5. K. Yoshida [J. Oceanogr. Soc. Jap. 15, 159 (1959)] first derived Eqs. 2 and the solutions in Fig. 1.
6. A. E. Gill (preprint) discusses a model for the westward surface currents and eastward undercurrent which are found in the Pacific and Atlantic oceans. The extension to this Indian Ocean case is trivial; our contribution here is the numerical solution. G. Philander [Rev. Geophys., 11, 513 (1973)] has reviewed the evidence for the westward-flowing equatorial jet found in the equatorial Pacific and Atlantic.
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