Rotary Wind Spectra in a Sea Breeze Regime

JAMES J. O'BRIEN
Dept. of Meteorology and Oceanography, Florida State University, Tallahassee 32306

R. DALE PILSBURY
School of Oceanography, Oregon State University, Corvallis 97331
1. Introduction

This note is designed to introduce to meteorologists a relatively new descriptive technique, the rotary spectrum, which is rapidly becoming a standard tool in physical oceanography. Horizontal wind time series, obtained along and off the coast of Oregon in an intense sea breeze regime, are used to illustrate the usefulness of this new technique in atmospheric science. During the summers of 1972 and 1973, two major air-sea interaction experiments, designated CUE-I and CUE-II (Coastal Upwelling Experiment), were conducted, each collecting large amounts of atmospheric and oceanographic data in order to study processes in a coastal upwelling environment (O'Brien and Tamura, 1972; Pillsbury and O'Brien, 1973).

The rotary spectrum is a representation in frequency space of the variance spectrum of a two-dimensional (vector) time series. The variance for each frequency band is divided into two components which are interpreted as the clockwise rotating variance (at negative frequency) and the counterclockwise rotating variance.
(at positive frequency). The analyzed time series may be reconstructed in a new coordinate system which depends on frequency such that the cospectrum (in-phase component) is theoretically zero for all frequencies. Since the rotary spectrum for a vector time series is invariant under a coordinate rotation, it is a fundamental frequency representation of the time series. In oceanography, for example, considerable variance is usually detected in a frequency band at the local inertial period. Since the inertial oscillations must induce a clockwise-rotating hodograph, the variance in the rotary spectrum for the inertial frequency will appear as a peak at the negative frequency corresponding to the local inertial period.

In this explanatory note, we briefly describe the analysis technique and present some sea-breeze rotary spectra.

2. The rotary spectrum

Any band-limited series of horizontal velocity can be considered as a realization of a continuous, stationary, stochastic process with a zero mean and constant variance. Fast Fourier transform algorithms may be used to partition the sample variance among frequency (or wavenumber) bands. Consider that a typical realization for an arbitrary frequency band, $f \pm \Delta f$, for the horizontal velocity $\mathbf{w}$ is

$$\mathbf{w} = \mathbf{u} + \mathbf{i}\mathbf{v},$$

where the components $\mathbf{u}$ and $\mathbf{v}$ are arbitrary sinusoids

$$u(t) = a_1(f) \cos(2\pi ft) + b_1(f) \sin(2\pi ft),$$
$$v(t) = a_2(f) \cos(2\pi ft) + b_2(f) \sin(2\pi ft).$$

The variance partition of an arbitrary sinusoid into two components can be easily calculated. If we replace $\cos(2\pi ft)$ and $\sin(2\pi ft)$ by complex exponentials, then

![Figure 2. Time series of horizontal wind components at the south jetty at Newport, Ore., 0900 GMT 1 August 1972 to 0800 GMT 1 September 1972. Position is 44°33.4'N, 124°03.9'W, 10 m above sea level. Each tick is a new day at (XXX) GMT. The sea breeze is particularly evident in the $u$ series after 23 August.](image)

![Figure 3. Rotary spectrum for Newport winds for 31 days beginning 0900 GMT 1 August 1972. The large peak at ±0.04 cycle hr⁻¹ is due to the sea breeze.](image)
The rotary spectrum is derived (Gonella, 1972; Mooers, 1974) by considering a transformation of variables from the set \((a, b, \tau, b_2)\) to a set \((A, C, \eta, \bar{\eta})\) such that

\[ w = u + iv = A e^{i\eta} + C e^{-i\bar{\eta}}. \]

In the hodograph plane, \(A + C\) is the length of the semi-major axis of the ellipse, \(|A - C|\) the length of the semi-minor axis, \((\eta - \bar{\eta})/2\) the direction of the major axis, and \((\eta + \bar{\eta})/2\) the temporal phase of the ellipse (Mooers, 1974). The amplitude \(A\) corresponds to the counterclockwise component of the motion, while \(C\) corresponds to the clockwise component. The variance of \(w\) is \(2(A^2 + C^2)\) which has been partitioned as a function of frequency into counterclockwise and clockwise portions. The rotary spectrum is a plot of \(2TA^2\) vs frequency at positive frequency and \(2TC^2\) vs frequency at negative frequency (where \(T\) is the record length in time).

The transformation used by the analyst is

\[ A = \frac{1}{2} [ (b_2 + a_0)^2 + (a_2 - b_1)^2 ] \]
\[ C = \frac{1}{2} [ (b_2 - a_0)^2 + (a_2 + b_1)^2 ] \]
\[ \tan \eta = \frac{a_2 - b_1}{a_1 + b_1} \]
\[ \tan \bar{\eta} = \frac{a_2 + b_1}{b_2 - a_1} \]

Comparison of (3) with (2) indicates that \(A\) is the amplitude of \(w\) at positive frequency and \(C\) the amplitude...
tude of \( w \) at negative frequency. The squared modulus of the Fourier transform of \( w \) is proportional to the rotary spectrum. The reader may verify that the rotary spectrum is simply the sum of the kinetic energy (speed) spectrum plus twice the quadrature spectrum between \( u \) and \( v \). Since the speed spectrum is even with respect to frequency the “character” or shape of the rotary spectrum is determined by the quadrature (out-of-phase) spectrum.

The historical background for the rotary spectrum is somewhat difficult to trace. The hodograph ellipse is common in tidal analysis (Godin, 1972). Both Fofonoff (1969) and Mooers (1970) independently contributed to the development of the rotary spectrum. Gonella (1972) has used the rotary spectrum for analyzing the coupling between wind and surface currents. He also derives additional statistics relevant to the rotary spectrum not described here. Perkins (1970) and Pillsbury (1972) have used the technique. This list is certainly not complete. Mooers (1974) has prepared a definitive explanation, and, in particular, he has extended the technique to cross-spectrum analysis of a pair of horizontal velocity time series.

3. Sea breeze wind time series

The strong thermal contrast created by surface heating and oceanic coastal upwelling induces a consistent sea-land breeze circulation off the Oregon coast in summer. The thermal circulation is particularly evident during periods of winds favorable for upwelling. This has been documented recently by Johnson and O’Brien (1973). During the summer of 1972, the CUE-I team maintained several land-based anemometers and ocean buoys with anemometers off the coast. These data are available. Detailed analysis of each time series has been presented in atlas form (O’Brien et al., 1973). In this note we wish to present a few time series and rotary spectra taken during August 1972 to illustrate the usefulness of the new technique.

The CUE-I experimental area is shown in Fig. 1.
Anemometers were installed at six coastal stations and on nine ocean buoys. Results for August 1972 from two land and two ocean sites are presented. Fig. 2 shows the time record of $u$ and $v$ from the Newport anemometer. This site is on the Newport south jetty with excellent exposure to oceanic air flow. The winds were favorable (from the north) for upwelling in early August and after 22 August. In the latter period the persistent sea breeze can be easily observed in the data. The rotary spectrum for these data is shown in Fig. 3. The significant peaks at $\pm 0.04$ cycle hr$^{-1}$ represent the sea breeze contribution to variance. The large, broad peak at zero frequency is the "weather." There is some indication in Fig. 3 that the clockwise rotating contribution at $-0.04$ cycle hr$^{-1}$ is more than that at $+0.04$.

The land-based anemometer located near Depoe Bay also had excellent oceanic exposure. In Fig. 4 the time series is shown. It is qualitatively similar to Newport (Fig. 2). The rotary spectrum for Depoe Bay winds (Fig. 5) indicates a statistically significant peak at $-0.04$ cycles hr$^{-1}$. This indicates that the sea breeze circulation is predominantly a clockwise rotating oscillation. Hence, the Coriolis acceleration must be controlling this consistent "veering" of wind during the sea breeze events. In the dozen or so rotary spectra computed from CUE winds, the clockwise rotating component is dominant in almost all cases. The rotary spectrum easily permits this analysis.

The wind record for the NH-15 buoy is shown in Fig. 6. The time period for this record is a subset of that of the land-based anemometers. The rotary spectrum for NH-15 in Fig. 7 exhibits a peak at $\pm 0.04$ cycle hr$^{-1}$ with more variance in the clockwise (negative) peak. Thus, the sea breeze circulation influences the circulation at least 15 n mi offshore. The time series of the velocity components for the buoy at NH-20 in Fig. 8 is qualitatively similar to NH-15 (Fig. 6) for the similar period. (Note that the time period is different.) There appears to be a threshold problem for this anemometer. The rotary spectrum for these data from
NH-20 (Fig. 9) shows no significant peak at ±0.04 cycle hr⁻¹. We conclude that this is evidence that the surface manifestation of the sea breeze does not extend to 30 km from the shore off Oregon during this period.

4. Conclusion

The rotary spectrum technique has been briefly described. The reader should consult the references cited for more details. In the sea breeze regime off Oregon, the rotary spectrum of horizontal winds indicates that the sea breeze is predominantly a clockwise (veering) wind oscillation and that the sea breeze circulation at the ocean surface does not extend to 30 km offshore. The persistent clockwise-rotating energy in this 24-hr oscillation is expected to induce considerable clockwise-rotating energy in the diurnal period of the ocean velocity.

The rotary spectrum should be useful in other meteorological situations when evidence is required for the presence of clockwise or counterclockwise excursions in horizontal wind data due to standing or propagating waves. It is particularly useful for statistical analysis of vector time series when spectrum estimates which are independent of the orientation of the geographic coordinates are desired.

Acknowledgments. This is a contribution to the Coastal Upwelling Ecosystems Analysis Program (CUEA), a program of the International Decade for Ocean Exploration (IDOE) sponsored by the National Science Foundation under Grant GX-33502. Support for this work has been provided by the Office of Naval Research and the National Science Foundation (Grant GA-29734). We wish to thank John Kindle, Bill Gilbert, Bob Still, Joe Bottero and their colleagues for helping us collect and analyze these data. The anemometer at Depoe Bay was lent to us by the Facilities Division of NCAR. NCAR is sponsored by the National Science Foundation.

REFERENCES