The Influence of Bottom Topography on Upwelling off Peru

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ABSTRACT

The x-y-t, two-layer \( \beta \)-plane numerical model developed by Hurlburt (1974) is used to examine the upwelling system off Peru. The region off Peru from 14 to 15\( ^\circ \)S is one of strong and persistent upwelling. The most distinctive feature of the Peruvian upwelling circulation is a predominant poleward flow. A local area model, when forced by wind stress only, cannot account for the observed Peruvian circulation. When an additional barotropic forcing is applied in the model, a dominating poleward flow results. The effects of wind stress are felt on the upper layer and by the third day of integration an equatorward flow develops near the coast. Two functional representations of the actual topography are used in the model and compared to a flat-bottom case. Model results, when compared to observations, show that the observed upwelling maximum ~40 km south of 15\( ^\circ \)S is the result of a mesoscale topographic feature, a seamount. Variations of longshore and cross-shelf flow in cases with sloping topography are explained as a result of conservation of potential vorticity and conservation of mass transport over the seamount. Vertical cross sections of the model results show strong poleward flows with a narrow layer of equatorward flow near the coast. Vertical cross sections also reveal offshore flow in the upper 40 m below which a thicker onshore flow exists. As a result of the effects of rotation on the strong poleward flow, and the effect of a subgeostrophic lower layer, a narrow bottom offshore layer appears over the shelf.

1. Introduction

The need to study coastal upwelling from an ecological and climatic standpoint is well-known to scientists from many disciplines. From the viewpoint of the physical oceanographer, coastal upwelling is an important feature in the study of the eastern ocean circulation. During 1976–77, the field experiment, JOINT II, part of the Coastal Upwelling Ecosystems Analysis Program (CUEA), studied the upwelling region off Peru. The Peruvian coastal waters are some of the world’s most productive fishing areas. This is due to the fact that upwelling does not migrate up the coast as in other regions. Thus the Peruvian coast is dominated by strong and persistent upwelling. It is the nearshore circulation of this region that will serve as the topic of the present study.

Many investigators have realized the usefulness of numerical models in understanding the dynamics of coastal upwelling. The model of O’Brien and Hurlburt (1972) examined upwelling for the \( f \)-plane case. Hurlburt and Thompson (1973) extended the investigation to upwelling on a \( \beta \)-plane. In the numerical model of Hurlburt (1974), the effects of various types of coastline geometry and bottom topography were discussed. Peffley and O’Brien (1976) continued this line of study by incorporating into the Hurlburt model the coastline and topography of the Oregon coast.

The present study used the Hurlburt (1974) model to investigate upwelling off the coast of Peru. The upwelling circulation in this region appears to be quite different from that of Oregon and Northwest Africa. Smith (1978a) states “The response of the currents in the region off Northwest Africa and Oregon is closely coupled to the local wind—but not so off Peru where distantly caused phenomenon may dominate local effects.” Brink et al. (1978) have observed that though the wind stress in this region is to the northwest, the mean flow is generally toward the southeast. This flow was previously observed by Smith et al. (1971). Smith (1978b) suggests that fluctuations in the current propagating in the poleward direction may be baroclinic Kelvin waves, originating equatorward of 10\( ^\circ \)S and propagating past 15\( ^\circ \)S. Hurlburt et al. (1976) have suggested that poleward flow may be due to internal Kelvin waves originating at the equator and propagating poleward. The fact that fluctuations are not well correlated with the local winds suggests that a locally wind-driven system is not sufficient to explain the currents observed off Peru. The Hurlburt numerical model, used over a mesoscale region and driven only by wind stress, does not simulate the
flows observed off Peru. What sort of parameterization of the forcing is needed to obtain the observed poleward flow over the shelf?

Various theories have been proposed to explain the cause of a poleward flowing undercurrent. Garvine (1971) has shown the dynamic importance of the longshore pressure gradient to a steady-state upwelling model. However, he provides no physical mechanism for the production of this longshore pressure gradient. He does suggest that wind "piles up" the water in the direction of the wind stress. Hurlburt and Thompson (1973) have shown that the $\beta$ effect serves as a mechanism to induce a north-south sea surface slope. This, in turn, reduces the barotropic mode, allowing the formation of a lower layer poleward flow. Hurlburt and Thompson (1973) have shown that the $\beta$ effect serves as a mechanism to induce a north-south sea surface slope. This, in turn, reduces the barotropic mode, allowing the formation of a lower layer poleward flow.

In the following case study, we have chosen to use a barotropic forcing mechanism, in the form of an atmospheric pressure gradient, to parameterize the observed poleward flow over the shelf. Hurlburt (personal communication) has suggested that the atmospheric pressure gradient drives a persistent transport through this model because the flow through the open boundaries is not constrained by horizontal variations in the atmospheric pressure gradient or the effects of distant boundaries.

The present study is the first attempt, using the Hurlburt model, to study the upwelling circulation of a mesoscale region off Peru. Longshore varying bottom topography is included in this model. This topography is an idealized, functional representation of the actual topography. The region of coastal Peru between Pisco (14°S) and San Juan (15°20' S) was purposely chosen by the JOINT II investigators as the region of strongest and most persistent upwelling. By using an idealized topography, representing the major large-scale and mesoscale features in this region, the effects of these features on the upwelling circulation may be determined.

Several cases are examined in order to study the onset of coastal upwelling in a model with a Peru-like topography using a forced barotropic flow and the forcing due to wind stress. Two specific regions along the Peru coast are found favorable to upwelling. The location of these regions is determined by longshore topographic variations. A poleward flow is found in both upper and lower layers. Variations in the strength of this flow are affected by topography, latitude and wind stress. Continuous vertical cross-shelf sections are calculated using Ekman layers at the top, interface and the bottom. These reveal an onshore flow at mid-depths with an offshore flow in the upper layer and near the bottom on the shelf. Similar flows have been observed in the Peru upwelling region.

2. The model

The model, developed by Hurlburt (1974), is an $x$-$y$-$t$, two-layer, primitive equation model on a $\beta$-plane. Driving forces for the model are both the wind and the atmospheric pressure gradient which forces a barotropic poleward flow. The latter is new for upwelling models. The system of equations defining the model is nonlinear and the model retains the free surface.

A three-dimensional view of the model geometry is shown in Fig. 1. The Peruvian coast, in the area of concern, is oriented at approximately a 45° angle in reference to latitude and longitude. The model coordinate system, corresponding to this orientation, has also been rotated 45°. All future reference to direction is based on the rotated system. Thus true northwest is now the rotated basin's north. This rotation results in a simple and economical way to obtain high resolution near the coast. A right-handed Cartesian coordinate system is used. The origin is
determined by the intersection of the easternmost edge of the fluid, at a specific latitude, and a bottom reference level usually located at the greatest depth of the fluid.

The horizontal dimensions of the fluid are \( L_x \) and \( L_y \). Both the eastern and western boundaries are straight solid walls while the northern and southern boundaries are open. These open-ended boundary conditions are designed to allow the development of a Sverdrup interior in a basin of only mesoscale north-south extent. The retention of a Sverdrup interior is of dynamic importance to the upwelling system. The large-scale wind stress curl is small over the eastern side of oceans. The Sverdrup balance minimizes the strength of the wind-driven barotropic mode and permits a more realistic dominance of the baroclinic mode (Hurlburt and Thompson, 1973).

**a. The system of equations**

The fluid used is assumed hydrostatic, homogeneous and incompressible in each layer. Thermodynamics, as well as thermohaline and tidal effects, are neglected. Thompson (1974) has shown the importance of thermodynamics and thermohaline mixing when the interface between layers approaches the surface. When the interfacial displacement is small, the exclusion of thermodynamics and thermohaline effects from the model does not prove a significant problem. In this model, as in Hurlburt (1974), the interface surfaces in approximately six days. Thus, without including thermodynamic and thermohaline effects, solutions beyond day 5 may not be useful and are not included here. Peffley and O'Brien (1976), however, have shown that five days of integration are long enough to understand the effect of topography on local circulation.

The model equations are the vertically integrated primitive equations on a \( \beta \)-plane for a stably stratified, rotating ocean:

\[
\frac{\partial V_1}{\partial t} + V_1 \cdot \nabla V_1 + k \times f V_1 = -\frac{1}{\rho_1} \nabla P_a, \tag{1}
\]

\[
g \nabla (h_1 + h_2 + D) + \frac{\tau_z - \tau^*}{\rho_2} + A \nabla^2 V_1, \tag{2}
\]

\[
\frac{\partial V_2}{\partial t} + V_2 \cdot \nabla V_2 + k \times f V_2 - \nabla P_a - g \nabla (h_1 + h_2 + D) + g' \nabla h_1 \]

\[
\frac{\rho_1}{\rho_2} \nabla^2 V_2, \tag{3}
\]

\[
\frac{\partial h_1}{\partial t} + \nabla \cdot (h_1 V_1) = 0, \tag{4}
\]

where the subscripts 1 and 2 denote upper and lower layers, respectively. Other terms are defined as follows:

\[
V = u_i \mathbf{i} + v_i \mathbf{j}, \quad f(x, y) = f_0 + \beta(y - y_0) + \beta(x - x_0) \tag{5}
\]

\[
g' = g(\rho_2 - \rho_1)/\rho_2 \quad \tau_z = \tau_{z1} + \tau_{z2} \quad \tau_i = \rho C_i |V_1 - V_2|(V_1 - V_2) \quad \tau_b = \rho C_b |V_2| V_2 \]

where \( i = 1, 2 \) denotes layers. Symbols are defined in the Appendix.

The boundary conditions used in the model are no slip and kinematic for the eastern and western boundaries. A quasi-symmetric boundary condition, after Hurlburt and Thompson (1973), is used for the open boundaries. The boundary condition sets longshore derivatives equal to zero in Eqs. (1)-(4), except for the north-south pressure gradient, atmospheric pressure gradient and in the continuity equation.

Two major assumptions are required to determine the north-south pressure gradient. First, the \( v \) field must be in nearly geostrophic balance such that

\[
f v_1 = g \frac{\partial}{\partial x} (h_1 + h_2 + D) \]

\[
f v_2 = g \frac{\partial}{\partial x} (h_1 + h_2 + D) - g' \frac{\partial h_1}{\partial x} \]

Taking \( \partial / \partial y \) of these equations and integrating over \( x \), along with the fact that the north-south pressure gradient is set equal to zero at the western boundary and the atmospheric pressure gradient is constant across the basin, the equations are

\[
g \frac{\partial}{\partial y} (h_1 + h_2 + D) |_x \]

\[
= \left[ \int_{-L_x}^{x} f v_1' dx + \frac{\partial v_1'}{\partial y} \right]_{-L_x}^{x}, \tag{6}
\]

\[
= \left[ \int_{-L_x}^{x} f v_2' dx + \frac{\partial v_2'}{\partial y} \right]_{-L_x}^{x} \tag{7}
\]

Since the longshore derivatives of velocity are assumed small, with the exception of \( \partial v / \partial y \) in the continuity equation, a second assumption,

\[
\left| \int_{-L_x}^{x} f \frac{\partial v_1'}{\partial y} dx \right| \ll \left| \int_{-L_x}^{x} f v_1' dx \right|
\]

further simplifies Eqs. (6) and (7).
Thus \( V_1' \) and \( V_2' \) of Eqs. (6) and (7), representing the barotropic part of the velocity and the geostrophic part of the baroclinic velocity may be written in vector form as

\[
\begin{align*}
V_1' &= V_1 - V_A \\
V_2' &= V_2 + \frac{h_1}{h_2} V_A 
\end{align*}
\]  

(14)

The model is started from rest. The wind stress and pressure gradient forcing are applied impulsively at initial time. Incorporation of these boundary and initial conditions closes the system. The complexities of such a nonlinear system require numerical methods to obtain a solution.

b. Numerical formulation

The finite-differencing scheme, incorporated into the model by O’Brien and Hurlburt (1972) and Hurlburt (1974), is semi-implicit in the \( x \) direction and explicit in the \( y \) direction. Use of a semi-implicit scheme in the \( x \) direction allows a larger time step than that usually dictated by the Courant-Friedrichs-Lewy (CFL) linear stability condition. Use of an explicit scheme in the \( y \) direction places the approximate restriction on the time step

\[
\Delta t \leq \frac{(\Delta y)_{\text{min}}}{[g(H_1 + H_2)_{\text{max}}]^{1/2}} 
\]  

(15)

given by the CFL condition. From the economic standpoint, a limit must be placed on the basin depth and the fine resolution of longshore scales, in spite of the desire for high resolution, small \( \Delta y \), and a realistically deep basin.

Other properties of the finite-differencing scheme are use of leapfrog time differencing for the Coriolis and nonlinear terms and use of the Crank-Nicholson (1947) scheme, which treats the diffusive terms semi-implicitly in the \( x \) direction. Other frictional terms are lagged in time. Advective terms are handled by scheme F from Grammelvtedt (1969).

Reduction of the number of grid points is accomplished by using a variable resolution grid. The \( \Delta x \) used in the model is a discrete function of \( x \). In the eastern most 40 km, the value of \( \Delta x \) is 2 km. Moving westward, \( \Delta x \) takes on values of 4, 10, 40 and 100 km. In the \( y \) direction, an analytically stretched variable, defined by Schulman (1970), was used. Implementation of this stretched coordinate approach gives maximum resolution of \( \Delta y = 4.1 \) km in the center of the basin and minimum resolution of \( \Delta y = 13.1 \) km at the northern and southern boundaries.

Hurlburt and Thompson (1973) have shown that the longshore flow and upwelling circulation are independent of basin width, if the basin is wide enough to permit the development of a Sverdrup
The width of the present model is 1500 km so that western boundary circulations have no effect on the eastern boundary and longshore flows are independent of basin width. In the east-west direction, a $\Delta x < 5$ km is required to resolve the boundary layers. Thus a fine grid is used in the eastern portion of the basin and a coarse grid in the western part of the basin where solutions are relatively unimportant to this study.

### Cross-shelf physics

One of the assumptions used in the model is that Ekman dynamics is important to the vertical structure of the flow field. Ekman-like boundary layers are expected and observed near the surface, pycnocline and bottom, all regions of large vertical shear. It will be of interest, then, to look at a vertical cross-shelf section of the flow generated by the model. This cross-sectional analysis is done using a method developed by Thompson (1974) in which analytical solutions are obtained for the departures from the vertical mean of the velocity fields. Boundary layers are thus introduced near the internal and external fluid interfaces. The simple upwelling circulation obtained from the vertically averaged equations is altered by the mass flux into and out of these boundary layers. If $V'_l$ is the departure from the mean velocity within each layer, then the Ekman equations are

$$k \times fV'_j = A_e \frac{\partial V'_j}{\partial Z^2}, \quad j = 1, 2.$$  

If the sea surface is designated by $z = 0$, the pycnocline by $z = b$ and the bottom by $z = c$, the statement of the problem becomes

$$\frac{\partial^2 w_1}{\partial Z^2} - s^2 w_1 = 0, \quad b < z < 0$$

$$\frac{\partial^2 w_2}{\partial Z^2} - s^2 w_2 = 0, \quad c < z < b$$

where

$$s^2 = \frac{if}{A_e},$$

$$w_1 = u'_1 + iv'_1,$$

$$w_2 = u'_2 + iv'_2.$$

We also require the integral constraint that, when vertically integrated over the layer, $w_j$ is zero. Four
features as Cabo Nazca, Punta Santa Ana, Punta San Nicholas and Punta San Juan. Peffley and O'Brien (1976), however, have shown that coastline features are not as influential as topographic variations in determining the longshore upwelling circulation. Observations of the coastline from Fig. 2 show it has a general orientation of 45° to latitude and longitude.

The bottom contours used in the cases examined here are functional estimates of the actual topography, representing the most distinctive topographic features. The first set of bottom contours, shown in Fig. 3, exhibits two distinctive topographic features distinguishing the northern from the southern region of interest. The first is the broad, flatish shelf and gently tilting continental slope in the north and the second, the steepening of both these slopes below 15°S.

The function used to obtain this idealized topography was a hyperbolic tangent function of the form

\[
h(x,y) = q(y) \left[ ax + \tanh \left( \frac{x - q(y)}{\sigma} \right) + b(y) \right], \quad (19)
\]

where \( q \) is also a hyperbolic tangent represented by

\[
q(y) = k \left[ \tanh \left( \frac{y + y'}{\sigma'} \right) + b' \right],
\]

such that

\[
h(0,y) = 74000 = H,
\]

\[
\lim_{x \to \pm} h(x,y) = 0,
\]

such that

\[
b(y) = -\alpha q(y) = \tanh \left[ \frac{1 - \gamma q(y)}{\sigma} \right]
\]

\[
c(y) = \frac{H}{-\alpha q(y) - \tanh \left[ \frac{1 - \gamma q(y)}{\sigma} \right] + \tanh \left[ -\gamma q(y) \right]}
\]

This hyperbolic tangent topography follows the actual topography closely. The maximum depth of 800 m is located 35 km from the coast in the north and 14 km in the south, corresponding to the actual topography. The functional estimate of the shelf extends to 15 km offshore in the north and 5 km offshore in the south, again a good representation of the real topography. The hyperbolic tangent function proves to be a good representation of these two important features of the topography.

The second set of contours is shown in Fig. 4. This case incorporates the mesoscale seamount into the previous idealized topography. The function used to estimate the seamount was an ellipse of the form
\[
\left[ c^2 - \frac{c^2}{a^2} (x - x_0)^2 \right]^{1/2} + z_0, \quad (20)
\]

with \( z \) representing height above the reference level depth 800 m and the center of the seamount located at
\[ x_0 = 8 \text{ km}, \quad y_0 = -41 \text{ km}. \]

The ellipse was incorporated into the previous topography by using the ellipse value whenever it proved to be larger than the original hyperbolic tangent value. A 1-2-1 running average in the \( y \) direction was applied once in order to blend smoothly the two functions. Due to the fine resolution in \( x \), smoothing is not required.

The functional representation of the seamount differs only slightly from the actual topography. The ellipse extends from 17 to 67 km south of 15°S and 36 km offshore. The actual seamount extends from ~20 to 60 km south of 15°S and 36 km offshore.

In both of the topographic representations described above, the values of the isobaths in the northernmost 50 km and southernmost 50 km of the basin are extensions of the isobath values at \( y = 100 \) km and \( y = -100 \) km, respectively. These regions of \( y \)-independent topography have been included to ensure that derivatives in \( y \) be small near the boundaries, as required by the boundary condition.

The topography with and without the seamount has been used in the model as representative of the major features of the actual topography. Flat bottom model cases have also been used in this study.

### Table: Model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>East-west basin extent</td>
<td>( L_e )</td>
<td>1500 km</td>
</tr>
<tr>
<td>North-south basin extent</td>
<td>( L_n )</td>
<td>300 km</td>
</tr>
<tr>
<td>Maximum basin depth</td>
<td>( H_e + H_n ) max</td>
<td>800 m</td>
</tr>
<tr>
<td>Initial upper layer thickness</td>
<td>( H_u )</td>
<td>50 m</td>
</tr>
<tr>
<td>Initial lower layer thickness</td>
<td>( H_l )</td>
<td>variable</td>
</tr>
<tr>
<td>Coriolis parameter at 15°S latitude</td>
<td>( f _0 )</td>
<td>( -0.3764 \times 10^{-4} ) s^{-1}</td>
</tr>
<tr>
<td>Horizontal eddy viscosity coeff.</td>
<td>( A )</td>
<td>( 5 \times 10^4 ) m^{2} s^{-1}</td>
</tr>
<tr>
<td>Vertical eddy viscosity coeff.</td>
<td>( A_v )</td>
<td>( 5 \times 10^4 ) m^{2} s^{-1}</td>
</tr>
<tr>
<td>Upper layer density</td>
<td>( p_u )</td>
<td>( 1.0 \times 10^3 ) kg m^{-3}</td>
</tr>
<tr>
<td>Lower layer density</td>
<td>( p_l )</td>
<td>( 1.002 \times 10^3 ) kg m^{-3}</td>
</tr>
<tr>
<td>Gravitational acceleration</td>
<td>( g )</td>
<td>( 9.8 ) m s^{-2}</td>
</tr>
<tr>
<td>Reduced gravity</td>
<td>( k' )</td>
<td>( 2 \times 10^{-1} ) m s^{-1}</td>
</tr>
<tr>
<td>Interfacial drag coefficient</td>
<td>( C_l )</td>
<td>( 1 \times 10^{-4} )</td>
</tr>
<tr>
<td>Bottom frictional drag coefficient</td>
<td>( C_b )</td>
<td>( 1 \times 10^{-4} )</td>
</tr>
<tr>
<td>Longshore pressure gradient</td>
<td>( \frac{\partial P}{\partial y} )</td>
<td>( 1.25 \times 10^{-3} ) N m^{-4}</td>
</tr>
<tr>
<td>Time step</td>
<td>( \Delta t )</td>
<td>36 s</td>
</tr>
<tr>
<td>Grid increment in ( x ) direction</td>
<td>( \Delta x )</td>
<td>variable</td>
</tr>
<tr>
<td>Grid increment in ( y ) direction</td>
<td>( \Delta y )</td>
<td>analytically stretched variable</td>
</tr>
</tbody>
</table>

### 3. Case descriptions

A number of case studies incorporating the two idealized Peru bottom topographies and flat bottom were made. The parameter values given in Table 1 were common to all cases.

The longshore extent of the basin, the area 150 km northwest and 150 km southeast of the intersection of 15°S with the Peruvian coast, contains the area of interest to the JOINT II experiments. Though the forcing of the flow in this area is suggested to be a macroscale phenomenon, the region of interest to the JOINT II study is only of mesoscale extent. In order to make a more detailed study of the characteristics of upwelling circulation in this limited region and for economic reasons, a mesoscale basin was chosen over one of larger longshore extent.

There was a good deal of freedom in choosing the cross-shelf extent of the basin. As stated previously, Hurlburt and Thompson (1973) have shown that if the basin is greater than 1000 km wide, the upwelling circulation in the model is not affected by the western boundary solution. For this model, the actual area of interest lies in the first 100 km from shore so that a basin width of 1500 km was chosen. The vertical extent of the basin was chosen to be 800 m. This is deeper than the expected maximum upwelling depth but was chosen to include flow over the continental slope and still retain an economical time step. Solutions using basins of this depth differ little qualitatively with a basin of depth \( O(200 \text{ m}) \) as observed by Thompson (1974).

The permanent pycnocline is represented by the layer interface. Upwelling is signified by the dis-
chosen to be $10^{-5}$ after Thompson and O'Brien (1973). The bottom stress value of $10^{-5}$ was suggested by Proudman (1953). The value of $\Delta t$ was chosen to be $\sim 80\%$ of the value dictated by the CFL condition.

The wind stress, shown in Fig. 5, is chosen to have zero curl in the upwelling region. The wind stress component $\tau_{x\tau}$ in the $x$ direction was set equal to zero while $\tau_{y\tau}$ attains a maximum value of 0.5 N m$^{-2}$ in the upwelling region. This value is in agreement with observations by Smith (1978a).

The value of the atmospheric pressure gradient was chosen so that it produced values for the poleward flow in the model, similar to those observed. If pressure gradient is replaced by atmospheric pressure gradient, Eq. (18) from Hurlburt and Thompson (1973) give the relationship

$$\frac{\partial P_A}{\partial y} = \rho \beta V L,$$

assuming the value of the atmospheric pressure gradient to be zero at the western boundary. This produces a value of order $10^{-2}$ N m$^{-3}$ for $\partial P_A/\partial y$. The value $1.25 \times 10^{-2}$ N m$^{-3}$ was chosen for the model. When comparing this value to observed values, it is slightly high. An average pressure change taken from 1977 surface pressure maps is 7 mb change over 10$^\circ$ of latitude. Using that value as an average, the atmospheric pressure gradient over 300 km is $2.1 \times 10^{-3}$ N m$^{-3}$. The value used in the model was chosen as the value best parameterizing the observed flow.

Three cases will be discussed in this paper. All cases were started from rest and driven by the wind stress of Fig. 5, and the atmospheric pressure gradient. Each case has a straight coast and a basin depth of 800 m. Case I is a flat bottom case. Case II uses the topography represented by Fig. 3 and case III uses the topography of Fig. 4. All cases were integrated for five days with wind stress and pressure gradient values remaining unchanged.

4. Results

Many case studies were made using the model described in Section 2. The results of these cases were studied by comparing five different fields determined by the model: the upper layer zonal velocity, upper layer meridional velocity, lower layer zonal velocity, lower layer meridional velocity and pycnocline height anomaly. By observing the similarities and differences in the various cases, new insights into the dynamics of upwelling may be obtained.

a. Dynamics

Although the basic dynamics of upwelling on a $\beta$ plane has been previously discussed by Hurlburt
and Thompson (1973), a short review will be beneficial to the understanding of this paper. For a flat-bottom case with an equatorward wind stress, an equatorward surface jet and poleward undercurrent appear. The jet has been explained by a conservation of potential vorticity argument. The poleward flow is explained in terms of matching a near-zero Sverdrup interior to an upwelling boundary layer. In the upper layer, the offshore flow is weaker than the Ekman drift due to a north-south sea surface slope. In the region outside the upwelling zone, onshore flow in the lower layer is balanced geostrophically. Inside the upwelling region, geostrophy and Ekman drift break down to satisfy the boundary condition $u = 0$ at the coast. Thus an equatorward acceleration appears in the upper layer and a poleward acceleration in the lower layer.

This study is particularly concerned with incorporating a useful parameterization of the poleward undercurrent. A poleward undercurrent has been found when the longshore pressure gradient reduces the barotropic mode near the coast. Hurlburt and Thompson (1973) have shown that a north-south sea surface slope may be induced by the β effect, i.e.,

$$\frac{\partial}{\partial t} \left( \frac{\partial v_1}{\partial x} \right) + \frac{\partial}{\partial x} \left( \frac{u_1}{\partial x} \right) + f \frac{\partial u_1}{\partial y} + \frac{\partial f}{\partial y} v_1,$$

$$+ \frac{\partial f}{\partial x} u_1 = \frac{\partial}{\partial x} \left( \frac{\tau_{xy} - \tau_{iy}}{\rho_l h_1} \right) + A \frac{\partial^2 v_1}{\partial x^2}. \quad (23)$$

Upon integration of this equation and the y-momentum equation and recalling the fact that the pressure gradient is set equal to zero at the western boundary and allowing that the atmospheric pressure gradient is constant in $x$, the following relationship is obtained:

$$\int_{-L_x}^{x} \beta(v_1) dx = g \frac{\partial}{\partial y} (h_1 + h_2 + D) \bigg|_{x}. \quad (24)$$

Thus, a north-south pressure gradient is induced by the β effect. In the lower layer the following relationship exists:

$$\int_{-L_x}^{x} \beta(v_2) dx$$

$$= g \frac{\partial}{\partial y} (h_1 + h_2 + D) \bigg|_{x} - g' \frac{\partial h_1}{\partial y}.$$  

Initially, the north-south pressure gradient and atmospheric pressure gradient dominate both layers. After three days of integration, the effects of the wind stress are seen in the upper layer, causing a weakening of the poleward flow and finally an equatorward flow near the coast. Test cases made on the topography of case III using atmospheric pressure gradient forcing alone and then atmospheric pressure gradient and wind stress forcing of Fig. 5, show a more rapid weakening of the flow near the coast in the case with wind stress. Test cases run with a wind stress of 1 dyn cm$^{-2}$ and a slightly larger pressure gradient, show a much quicker decrease in the poleward flow with time resulting from the increase in wind stress. A test case was also made with a reverse atmospheric pressure gradient, showing flows entirely equatorward in both the upper and lower layers.

It is of utmost importance to note that when the model was driven by wind stress alone, only a very weak poleward undercurrent appeared. Thus another mechanism, artificial in nature, is introduced to drive a deep poleward flow. A barotropic pressure gradient serves this purpose.

b. Upper layer flow

Study of the upper layer cross-shelf flow reveals quantitative similarities in all three cases. It should be noted here that the solutions at the northern and southern boundaries, in all of the subsequent figures shown in this paper, have been excluded. The region
The major longshore variation in $u_1$ is a result of the topography of the seamount. Over the relatively flat top of the seamount, offshore flow decreases. North of the steep seamount seen in Fig. 6a.

of particular interest to this study lies, at most, 100 km to the northwest and southeast of the intersection of 15°S and the coast. The topography has been designed in such a way that the northernmost and southernmost 50 km of the basin are of an artificial nature. Therefore, the solutions in this region, which may contain some effects due to the model rotation are, by design, not of importance to this study. Thus the subsequent figures will range from $y = -100$ km to $y = 100$ km only. The cross-shelf extent of the basin used in the figures will be 100 km from the coast. This region includes the cross-shelf extent of the sloping topography and the area of interest to the upwelling system.

Upper layer flows in all cases are offshore, increasing in magnitude outward into the basin as seen in Fig. 6a.

**Fig. 6.** (a) The cross-shelf component $u_1$ of the upper layer flow for Case III and (b) the longshore component $v_1$ of the upper layer for case I, both in cm s$^{-1}$. These figures represent the flow after five days of integration. The following convention will be used in this and subsequent figures; dashed lines represent negative contours, solid lines represent positive contours.

**Fig. 7.** Cross-shelf flows $u_z$ for Cases I (a), II (b) and III (c). Values are in cm s$^{-1}$ and represent the fifth day of integration.
topography, offshore flow weakens becoming stronger again south of the seamount. This pattern is opposite that seen in the lower layer cross-shelf flow. Thus the upper layer attempts to balance the lower layer transport. The effects of the seamount are seen at least 50 km into the basin. Peffley and O'Brien (1976) concluded that the longshore dependence of $u_1$ is dominated by coastline variations. Aside from that due to the seamount, little longshore variation is seen in $u_1$ since the coastline is straight.

Upper layer cross-shelf velocity, as predicted by Ekman dynamics, is

$$u_1 = -\frac{\tau_{mn}}{\rho_1 h_1 f} = -0.027 \text{ m s}^{-1},$$

in good agreement with model values. Data taken during JOINT II from "C-line" buoys PS and PSS located 12 and 4 km from the coast, respectively, show an average $u$ value of $-0.028 \text{ m s}^{-1}$ at 20 m depth (Halpern, personal communication).

The longshore flow in the upper layer shows similar patterns for all three cases. Initially, all flows are poleward, increasing in magnitude away from the coast. In both topography cases, flows are slightly larger. As expected, topographic features have a smaller effect on the upper layer longshore flow than on the lower layer longshore flow. As mentioned in Peffley and O'Brien (1976), the close similarity in the longshore velocity fields between a flat bottom and topography case is not due to a lack of effect by topography; rather, a weaker cross-shelf baroclinic pressure gradient is compensated by a stronger barotropic mode in the topography case. The intensity of the poleward flow, in all three cases, is decreasing with time as the effects of the wind stress are more strongly felt. At day 5 (Fig. 6b) a small equatorward flow appears at the coast in all cases, though only case I is shown here.

c. Lower layer flow

The lower layer cross-shelf flow shows important differences among the three cases. Figs. 7a–7c represent the lower layer cross-shelf flow at day 5 for all three cases. In the flat bottom case, the flow is offshore with an extremely weak onshore flow near the coast. The flow exhibits a uniform pattern in the longshore direction. As expected, because the lower layer is thick, the flow is weaker than in the upper layer.

Case II also shows a weaker offshore flow in the western portion of the basin than in the upper layer. The most noticeable difference between this case and the flat-bottom case is a distinctive onshore flow near the coast. Fig. 7b clearly shows that topography affects the longshore variations in $u_2$. The difference between these cross-shelf flows may be explained in terms of mass continuity and a rising topography. From the $y$ momentum equation (22b)
maxima of onshore flow appear; one in the same location as the maximum of case II, and one further south, with a small region of offshore flow between them. These patterns, caused by topographic variations, are visible beyond the cross-shelf extent of the topography.

Before explaining the basic dynamics behind such a pattern of cross-shelf flow, it will be useful to observe the longshore flow fields for the three cases. Figs. 8a–8c show the flows to be poleward in all three cases. This poleward flow has been documented and discussed by Smith (1978b) and Brink et al. (1978). It is also true that the poleward flow in the lower layer in all three cases is larger than in the upper layer. The upper layer feels the effects of the wind stress far greater than the lower layer, as expected.

In Figs. 6b and 8, flat-bottom cases, a distinctive variation in the longshore flow appears. Flow is larger in magnitude in the northern portion of the basin. This pattern may be an effect of the rotated basin. If the basin was not rotated, an important scale would be the distance to the wind curl region. In the rotated basin, the distance from the coastal boundary westward to the open boundary is greater as we move from the northeast corner to the southeast corner of the coastal boundary. Thus external Rossby waves appear to propagate away from the coast faster at the southeastern portion of the boundary. Since

$$\text{FIG. 9. Contours of the pycnocline height anomaly at day 5 for cases I (a) and II (b). Contour values are in meters.}$$

$$\frac{\partial v_2}{\partial t} + fu_2 = -\frac{1}{\rho_s} \frac{\partial P_e}{\partial y}$$

$$-g \frac{\partial}{\partial y} (h_1 + h_2 + D) + g' \frac{\partial h_1}{\partial y} - \frac{\tau_{by}}{\rho_s h_2},$$

and the kinematic boundary condition, \(u_z = 0\) at the coast, \(\partial v_2/\partial t > 0\) for the flat-bottom case. In the cross-shelf sloping topography case, the interior transport and offshore transport over the shelf are approximately equal. Thus the onshore flow in the lower layer is required to become supergeostrophic by mass continuity forcing, i.e., \(\partial v_2/\partial t > 0\). And since \(f < 0\),

$$\left| \frac{\partial v_2}{\partial t} \right|_{\text{flat bottom}} > \left| \frac{\partial v_2}{\partial t} \right|_{\text{topography}};$$

therefore, \(v_2\) increases more quickly with time in the topography case until a frictional balance is achieved.

The cross-shelf flow in case III reveals features similar to those in case II. Here, however, two

$$\text{where positive } x \text{ is true eastward and positive } y \text{ is true northward and } gh_v \bigg|_{x_0} \neq 0.$$

Thus the assumption made previously,

$$\int_{-L_2}^{x} f v_2 dx < \int_{-L_2}^{x} \beta v dx,$$

which was used to calculate \(h_v\) at the open boundary, may not be as valid in a rotated basin as in an unrotated basin.

In the northern portion of the basin, the longshore flows in all three cases are quite similar with a slightly more poleward tendency in the topography cases. However, the longshore flows in cases II and III deviate from that of case I in the region of large longshore isobathic gradients. Case II exhibits larger poleward flow in this region and south of the region than in case I. A comparison of case III to case I shows an increase, a decrease and then another increase in poleward flow when looking from the northern to the southern portion of the basin. It is concluded then that these flows are topographically induced.

The explanation for these topographically induced variations in both the cross-shelf and longshore flow
fields is based on conservation of potential vorticity and conservation of mass transport over the seamount. The region of large longshore isobatic gradients in case II is a region of maximum onshore flow and decreasing longshore flow. These are the expected characteristics of flow following the isobaths and conserving potential vorticity. Case III shows similar flow patterns in the hyperbolic tangent region.

Two explanations may be given for the alongshore variations in both cross-shelf and longshore flow near the seamount. In this region as well as the hyperbolic tangent region, flow follows the pattern of the isobaths. In the northern half of the seamount, where isobaths turn toward the interior of the basin, flow decreases in the onshore direction and increases in the poleward direction. When the isobaths turn toward the coast on the southern half of the seamount, flow increases in the onshore direction and decreases in the poleward direction. These large alongshore variations may also be explained as due to a conservation of mass transport. North of the seamount, the lower layer depth $h_2$ increases. Flow in this region slows down, becoming smaller in magnitude in both onshore and poleward flow. Rising over the seamount, where lower layer depth $h_2$ decreases, the speed of the flow increases in both the onshore and poleward directions. When the layer thickens south of the seamount, speeds decrease once again.

One final general feature of the cross-shelf flow is apparent in all three cases; the cross-shelf extent of the onshore flow is larger in the northern portion of the basin than the southern portion of the basin. This characteristic may be explained simply as due to the effects of geostrophy on the lower layer cross-shelf flow. As stated in the discussion of the dynamics on a $\beta$ plane, $u_2$ is in geostrophic balance. Thus $u_2$ varies inversely with $f$ and would be expected to increase when approaching the equator. Another possible physical explanation is that inertial oscillations excited in phase at different latitudes do not remain in phase with time (Hurlburt, 1974).

After seeing the results of the model forced with wind stress and a poleward lower layer flow, it is important to distinguish the separate effects of the two forcing mechanisms. Using the topography of case III, the model was forced separately by wind stress and then the barotropic poleward flow. From the results of these cases, it seems that the combined forcing case is a linear superposition of the solution of the separate forcing cases.

In the upper layer, the effects of wind stress dominate the cross-shelf flow. The offshore flow is characteristic of the wind stress case. The thickness of this flow lessens near the coast as a result of the barotropic forcing which causes a slight onshore flow near the coast. The upper layer longshore flow is dominated by the barotropic forcing effects. This case shows a small equatorward flow near the coast and poleward flow out into the basin. The wind stress case revealed only equatorward flow. Longshore variations in both cross shelf and longshore flow appear as effects of the barotropic forcing.

Both longshore and cross-shelf flows in the lower layer are dominated by the barotropic forcing. The results of that case bear a striking resemblance to the combined case. The model shows that both longshore and cross-shelf flows in the wind stress only case are very weak. Each case revealed longshore variations in the flow in the broad shelf and seamount regions, similar to the combined case. Thus a superposition of the two forcing mechanisms would reveal a case dominated by the barotropic poleward flow.

After the initial adjustment period of perhaps 1½ days, the flows change less dramatically. The flow in the wind-stress case increases very slowly, while the flow in the barotropic forcing case decreases slowly. This poleward flow, driven by the imposed barotropic pressure gradient will diminish slowly at the eastern boundary and become part of the western boundary layer. However, it remains an effective forced flow throughout the period of interest in the present study. The solutions do not oscillate over the five-day integration.

The general longshore pattern in the pycnocline height anomaly field, to be discussed in the next section, is attributed to the barotropic forcing. This is to be expected since this case dominates the lower layer flow patterns. Thus, even though the solutions do appear to be a superposition of both cases, the barotropic forcing seems to have the dominant effect in this model.
d. Upwelling

It is of utmost importance to this study to note the differences in amounts of upwelling due to the different topographic features. Figs. 9a and 9b show the upwelling patterns for cases I and II. The amounts of upwelling are also referred to as the pycnocline height anomaly, which describes the displacement of the layer interface from its initial depth.
The flat bottom case shows a simple upwelling pattern with increasing amounts of upwelling near shore. By day 5, an average of 45 m of upwelling is seen at 20 km offshore with a maximum of 46 m closer to the coast. Upwelling is rather uniform along the coast. The uneven appearance of the contour spacing at -40 and 70 km offshore result from numerical rather than physical effects. These locations represent the region in which the grid spacing interval changes causing these very localized kinks in the solutions.

Case II does not reveal a uniform longshore pattern. Instead, a maximum now appears at $y = 50$ km, over the broad shelf region of the topography. This characteristic is apparent across the basin where amounts of upwelling are larger in the north than the south. Recall that in the region of the broad shelf, the cross-shelf flow was onshore and poleward flow larger than in the south. Thus the tendency observed in Peffley and O'Brien (1976), for regions of larger poleward flow to also be regions of larger upwelling and an increased barotropic mode, is also observed in this model. The amount of upwelling increases rapidly near $15^\circ$S and $25-15$ km offshore. This increase, seen in both Figs. 9b and 10, is located in the region of maximum onshore and poleward flow.

In both Figs. 9 and 10, the effects of the upwelling anomalies are seen out to 100 km. The baroclinic radius of deformation is $\sim 25.7$ km. The barotropic radius of deformation is of the order of $10^9$ km. As a result of the uplifting of the free surface, flows are dominated by the much larger scale, the barotropic radius of deformation. In contrast to that found off Oregon and Northwest Africa, none of the velocity observations off Peru show any coastal jet or similar feature in the flow to indicate a dominance of the internal radius of deformation (Brink et al., 1980).

Perhaps the features of most interest are shown
in Figs. 10 and 11. Fig. 10 is the amount of upwelling for day 5 obtained from the model for case III. Fig. 11 is a representation of mean sea surface temperatures in the JOINT II 1976 region of research from Nanney (1978). Regions of largest upwelling coincide with regions of coldest temperatures while upwelling minima coincide with warm areas. Case III shows regions of largest upwelling agree fairly well with the mean map. One maximum lies ~55 km north of 15°S, while the model predicts it at ~50 km. The second maximum lies ~33 km south of 15°S while the model predicts the maximum to be near 30 km south of 15°S. The model's upwelling minimum appears south of the observed warmest temperatures. The fact that these highs and lows of upwelling do not exactly coincide is due to the fact that the model result is an instantaneous picture from a very particular realization. The topography in case III, though a close estimate to the actual topography, is still idealized. The wind stress is also idealized since it remains a constant. Brink (personal communication) has suggested that longshore variations in the wind stress may be responsible for the appearance of certain upwelling patterns. This hypothesis may be checked in future model cases. Thus, similar but not exact results are expected.

Fig. 10 exhibits the maximum in the north as expected from the results of case II. The new feature is the addition of another maximum in the region of the seamount and a resulting relative minimum in the region between the two maxima. The minimum occurs in the region just below the 15°S line where the flow becomes less poleward. The highest values occur where the flow becomes more poleward. Perhaps a simpler way to view the
cause of these minima and maxima of upwelling is to think of the lower layer flow coming from the north and being forced to rise over the seamount, thus inducing more upwelling just north of the seamount and less upwelling south of the seamount.

From the results of cases II and III, favored areas of upwelling seem to be over the broad shelf and very slightly north of the seamount's center. Upwelling contours appear to parallel the isobaths in most regions. Also, the longshore and offshore scales both seem to be based on the scale of the topography. Thus, as in Peffley and O'Brien (1976), here it is the topographic variation that dramatically affects the upwelling pattern.

**X-Z cross-shelf section**

Cross-sectional analyses of the cross-shelf and longshore flows, along the C Line, for the JOINT II project were done by Van Leer (1979) from cyclosonde data. The C Line is a line perpendicular to the coast ~15 km south of Cabo Nazca, from which large amounts of data were taken. Figs. 12a and 12b show the longshore field in cross section based on cyclosonde data from Van Leer for 28 and 31 May 1976. Initially, the flow is all poleward, weak in the upper layer and increasing with depth and weakening over the shelf. As time goes on, the flow becomes less poleward near the coast, until finally, after five days, a strong equatorward flow is established in the region nearest the coast while poleward flow is found only further out on the shelf.

The model shows quite similar results. Fig. 13a depicts a small equatorward flow in the upper layer near the coast with poleward flow below and further out into the basin. The magnitude of the poleward flow decreases rapidly toward the surface. Thus large velocity gradients are created in the upper layer indicative of the dominance of the baroclinic mode. As expected in a thick lower layer, the flow takes on barotropic features changing little with depth. As time goes on the equatorward flow is seen to strengthen and expand further into the basin (Fig. 13b). The magnitude of the longshore velocity weakens rapidly close to the shelf due to the boundary conditions. The observed data exhibits this same feature.

There is some discrepancy between the model and real data when considering the magnitudes of the flows. The intensity of the equatorward coastal flow in the model is approximately five times weaker than the observed flow. This could be partly explained by the low value of wind stress chosen for the model. Although the poleward flow also seems generally a bit weaker, the magnitudes of the longshore velocity in Fig. 13b may be slightly larger as the next day's data show a considerable drop in the magnitude to approximately ~10 cm s⁻¹.

Based on Van Leer's data, a cross section of the cross-shelf component of the velocity for 27 May 1976, is shown in Fig. 14. An offshore flow, which deepens away from the coast, is observed in the upper layer. Near 50 m depth, there is an onshore flow of larger magnitude. The onshore flow weakens approaching the shelf, and a weak offshore flow appears in a narrow region over the shelf.

The appearance of this offshore flow near the shelf has also been seen by O'Brien et al. (1980) in calculations done for an adjusted u field and vertical velocities. Fig. 15 is the calculation of the velocity fields for 11 April 1976, obtained using this adjusted u field. Note an offshore flow near the coast at 4, 12 and 20 km. Also note the thickening of the offshore layer with distance from the coast as observed by Van Leer. At ~40 m depth,
the flow becomes onshore and remains so until very near the shelf.

This reversal in flow at the bottom is due to an Ekman boundary layer. In the lower layer, the flow is nearly geostrophically balanced. Close to the bottom, viscous terms become large, reduce the Coriolis force and cause an offshore veering, or a reversal in the cross-shelf flow. The depth of this layer, from Ekman dynamics and using the value of vertical eddy viscosity chosen in this model, is \( \approx 16 \) m. This is in good agreement with the observed value from Fig. 14 which shows an average depth between 15 and 16 m.

Figs. 16a–16d show the cross-shelf velocity field derived from this model using the \( x-z \) cross-section method of Thompson previously described. In all
FIG. 16. Vertical cross section of the cross-shelf flow (cm s$^{-1}$) at day 5 obtained from model case III. (a) $y = -40$ km, (b) $y = 30$ km, (c) $y = 10$ km, (d) $y = 50$ km. The y location of each figure may be obtained by looking at Fig. 4.
four locations, the upper layer flow is offshore. It is interesting to note that the thickness of the layer increases away from the coast as seen in the actual data. This thickening of the upper layer offshore is due to the effects of the atmospheric pressure gradient as well as the upwelling. In test cases made using only atmospheric pressure gradient forcing, onshore flow appears near the coast, decreasing in value to ~25 km into the basin and then becoming offshore. This onshore component combined with the total offshore flow in the upper layer due to the wind stress causes the narrowing of the offshore layer near the coast. An offshore flow dominates the remainder of the picture except for a narrow region above the shelf. As expected from previous results discussed, the onshore flow near the sea-mount is larger than over the broad shelf as seen in Fig. 16a. At \( y = -30 \) km, the region of sharp changes in topography just north of the seamount, the flow is seen to be onshore very near the coast, but becomes offshore at distances ~20 km offshore. This flow corresponds to the region between the two onshore maxima of Fig. 10. Fig. 16c, at \( y = -10 \) km, is the approximate model location of the C Line for JOINT II. In the first 20 km from the coast, an offshore flow of maxima value (5 cm s\(^{-1}\)) is found in the upper 30 m. A weak offshore flow (<2 cm s\(^{-1}\)) appears in a layer 16 m thick above the shelf. These layer thicknesses correspond closely to Van Leer's data. The magnitudes of the observed offshore flow is <5 cm s\(^{-1}\) in both these layers, also close to the model values. The model onshore flow reaches a maximum of 2 cm s\(^{-1}\) in both these layers, also close to the model values. The model offshore flow is nonlinear and neglects thermodynamics. Two idealized bathymetries have been incorporated into the model. One, a hyperbolic tangent representation of the bottom topography, characterizes the broad shelf north of 15°S and narrow shelf south of 15°S. The second is an elliptical mound added to the hyperbolic tangent topography, simulating a mesoscale seamount. A barotropic flow, parameterized by introducing a constant atmospheric pressure gradient forcing, as well as a constant wind-stress forcing are used in this model.

Cases incorporating the previously mentioned idealized topography and forcing functions were compared. The model, using the hyperbolic tangent topography, revealed a maximum of upwelling over the broad shelf. With the addition of the seamount to the topography, a new upwelling maximum appeared just south of 15°S. This region has been documented by the JOINT II investigation as a region of intense upwelling. Specifically, this study suggests that this region of maximum upwelling is due to a mesoscale topographic feature, a seamount.

Topography is shown to affect the structure of both the upper layer and lower layer velocity fields. The seamount appears to be the most influential topographic feature on the longshore structure. Topographically induced variations in the lower layer longshore flow result in variations in the barotropic cross-shelf flow. The cross-shelf scale of the effects of the topography on these flows is larger than that of the topography itself.

In general, the patterns of longshore and cross-shelf velocity resulting from the model are in good agreement with observations. The vertical cross sections show an offshore upper layer of increasing thickness in the offshore direction, an onshore layer below and another offshore flow near the bottom due to Ekman dynamics. The magnitude of the flow fields is also in good agreement with observation. The cross-shelf velocity has been shown to be approximately equal to that observed. The longshore flow is generally larger than the observed flow. A reduction in the value chosen for the atmospheric pressure gradient would reduce the magnitude of these flows.

The method described in Hurlburt (1974) and Peffley and O'Brien (1976), that is, use of a wind-driven system, does not correctly model the observed upwelling system off Peru. Wind-stress forcing alone produced very weak poleward flow. The cause of the large poleward flow observed off Peru has been suggested to be a result of the propagation of large-scale baroclinic Kelvin waves originating near the equator. In order to make a detailed study of the mesoscale area of interest to JOINT II, a model of mesoscale longshore extent was used. Thus another mechanism to parameterize the remote forcing had to be introduced into this mesoscale system in order to obtain the strong poleward flow. The atmospheric pressure gradient, forcing a barotropic poleward flow, was chosen. Including this forcing along with the wind stress has been shown not only to induce a lower layer poleward flow, but also to induce flow patterns that agree with observations in both layers. With the addition of this mechanism, the model is able

5. Summary and conclusion

A two-layer, \( x-y-t, \beta \)-plane numerical model, developed by Hurlburt (1974), has been used to investigate upwelling features off Peru. The model is nonlinear and neglects thermodynamics. Two idealized bathymetries have been incorporated into the model. One, a hyperbolic tangent representation of the bottom topography, characterizes the broad shelf north of 15°S and narrow shelf south of 15°S. The second is an elliptical mound added to the hyperbolic tangent topography, simulating a mesoscale seamount. A barotropic flow, parameterized by introducing a constant atmospheric pressure gradient forcing, as well as a constant wind-stress forcing are used in this model.
to closely predict the location of the areas of maximum upwelling.

In summary, it must be remembered that this model is an idealized one designed to explain the effects of the bottom topography on upwelling off Peru. Due to a constant forcing by wind stress and the atmospheric pressure gradient, this model results in somewhat unrealistic features in the flows. Studies of the time-varying forcing should provide more realistic solutions. The exclusion of thermodynamics, which allows only five days of useful integration, should also be reconsidered when designing a more realistic model. A more detailed study of the instabilities caused by very large isobaric gradients may allow for the use of a smaller horizontal eddy viscosity. Incorporation of the actual topography should reveal the finer details of the longshore upwelling pattern. Finally, improvement on the solutions at the northern and southern boundaries could be made.

It may be concluded, then, that the choice of the atmospheric pressure gradient imposing a barotropic longshore flow as an additional means of forcing to the Hurlburt (1974) model, closely simulates the flows seen off the coast of Peru. It is also concluded that the two upwelling maxima observed are due to topographic effects, in particular the maximum observed south of 15°C is a result of the seamount, a mesoscale topographic feature.

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APPENDIX

List of Symbols

\[\begin{align*}
A & \quad \text{horizontal coefficient of eddy viscosity} \\
A_v & \quad \text{vertical coefficient of eddy viscosity} \\
C_f, C_b & \quad \text{drag coefficients for interfacial and bottom friction} \\
D(x,y) & \quad \text{height of the bottom topography above a reference level} \\
f & \quad \text{Coriolis parameter} \\
f_0 & \quad \text{Coriolis parameter at } y = 0 \\
g & \quad \text{acceleration of gravity} \\
g' & \quad \text{reduced gravity } [\frac{g(\rho_2 - \rho_1)}{\rho_2}] \\
h & \quad \text{total depth} \\
h_1, h_2 & \quad \text{instantaneous local layer thicknesses} \\
H_1, H_2 & \quad \text{initial layer thicknesses} \\
L_x, L_y & \quad \text{the dimensions of model in the } x \text{ and } y \text{ directions} \\
t & \quad \text{time} \\
u_1, u_2 & \quad x-\text{directed components of current velocity} \\
v_1, v_2 & \quad y-\text{directed components of current velocity} \\
x, y, z & \quad \text{Cartesian coordinates for a } 45^\circ \text{ rotated system}; x \text{ is positive north eastward, } y \text{ is positive northwestward, and } z \text{ is positive upward} \\
\beta & \quad \text{spatial gradient} \\
\Delta t & \quad \text{time increment in the numerical integration} \\
\Delta x, \Delta y & \quad \text{horizontal grid increment in } x \text{ and } y \\
\rho, \rho_1, \rho_2 & \quad \text{densities of sea water} \\
\sigma_i & \quad \text{(} \rho, t, 0 - 1 \text{) } \times 10^3 \text{, where } \rho, t, 0 \text{ is the sea water density corrected to atmospheric pressure} \\
\tau_{xy}, \tau_{xz}, \tau_{yz} & \quad x-\text{directed tangential stresses at the surface, interface and bottom} \\
\tau_{xy}, \tau_{yl}, \tau_{yv} & \quad y-\text{directed tangential stress at the surface, interface and bottom} \\
\end{align*}\]

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