A Coupled Ice-Ocean Model of Upwelling in the Marginal Ice Zone

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A dynamical coupled ice-ocean numerical model for the marginal ice zone (MIZ) is suggested and used to study upwelling dynamics in the MIZ. The nonlinear sea ice model has a variable ice concentration and includes internal ice stress. The model is forced by stresses on the air/ocean and air/ice surfaces. The main coupling between the ice and the ocean is in the form of an interfacial stress on the ice/ocean interface. The ocean model is a linear reduced gravity model. The wind stress exerted by the atmosphere on the ocean is proportional to the fraction of open water, while the interfacial stress ice/ocean is proportional to the concentration of ice. A new mechanism for ice edge upwelling is suggested based on a geostrophic equilibrium solution for the sea ice medium. The upwelling reported in previous models invoking a stationary ice cover is shown to be replaced by a weak downwelling due to the ice motion. Most of the upwelling dynamics can be understood by analysis of the divergence of the across ice edge upper ocean transport. On the basis of numerical model, an analytical model is suggested that reproduces most of the upwelling dynamics of the more complex numerical model.

1. INTRODUCTION

The aim of the present study is to investigate upwelling dynamics in the Marginal Ice Zone (MIZ) by means of a coupled ice-ocean model. The MIZ is a region connected to the ice edges of the world oceans. It may, somewhat arbitrarily, be defined as that region inward and outward from the ice boundary, which is significantly influenced by the presence of the ice edge. Typical MIZ conditions are found along the southern edges of the Greenland and Barents seas, in Baffin Bay, and along the complete northern edge of the Antarctic ice cover. An approximate position of the MIZ in the Greenland and Barents seas from March 14, 1978, is displayed in Figure 1.

Except for exploratory efforts in the past, the MIZ has only recently received systematically investigated. A review of our knowledge of both atmospheric, cryospheric and hydrospheric processes in the MIZ may be found in Andersen et al. [1980].

The MIZ has long been recognized to be a biologically productive region, where large numbers of marine birds and mammals congregate [Alexander, 1980]. As such, these regions therefore represent ecologically critical habitats in subpolar regions. One of the oceanic features known to provide regions of enhanced primary productivity is upwelling. Upwelling may be a frequent phenomenon in the MIZ. It has been observed by Buckley et al. [1979] in the MIZ north of Spitsbergen and also in the Bering Sea MIZ [e.g., Alexander and Niebauer, 1981]. Other interesting features in the MIZ include oceanic fronts, along ice edge oceanic jets, oceanic eddies, and vigorous heat exchange between atmosphere and ocean. An excellent review of the conditions in the MIZ in the Norwegian-Greenland Seas may be found in Wadhams [1981].

The possibility of upwelling at ice edges was first demonstrated by Gammelsrod et al. [1975] using a simple homogeneous model for the ocean. They parameterized the ice cover as free to move vertically, but unable to move horizontally. In their model the presence of the ice cover causes an abrupt change in the effect of the wind on the ocean surface. This change causes a divergence in the wind-driven oceanic circulation, which in turn causes the upwelling. Clarke [1978] extended their study to include stratification and parameterized the ice cover as a nonmoving rigid lid. As in Gammelsrod et al. [1975] he found that the wind stress curl caused by the presence of the ice cover was responsible for raising the pycnocline in a fashion very similar to the theories of coastal upwelling [O'Brien and Hurlburt, 1972; Gill and Clarke, 1974]. A numerical model of ice edge upwelling has recently been presented by Niebauer [1982] in which the ice cover was parameterized as a nonmoving rigid lid. Niebauer [1982] extended the earlier analytical works of Gammelsrod et al. [1975] and Clarke [1978] by introducing a melt-water stability or front close to the ice edge. His results confirm the earlier analytical works, but give a more realistic circulation in the ocean due to the introduction of melt water.

One of the most striking features observed and confirmed by satellite imagery is the mobility of the ice pack. For instance, Buckley et al. [1979] observed that the ice edge north of Spitsbergen in December 1977 was capable of moving several tens of kilometers in a period of few days. They also reported a pronounced upwelling, which they attributed to the wind stress, despite the fact that the winds were too weak to drive the observed upwelling while they were there. More recently, Pease and Muench [1981] observed the ice edge in the Bering Sea to move at a speed of 35 km day$^{-1}$ under gale conditions.

On this basis two intriguing questions arise. (1) Will the motion of the ice destroy or enhance the upwelling predicted by earlier models? (2) May the motion of the ice induce another physical mechanism for ice edge upwelling. More generally, one may ask how the presence of a moving low concentration ice cover affects the oceanic circulation in the MIZ (i.e., oceanic jets, fronts, eddies, etc.).

To study and possibly answer these questions and also to understand the oceanic and cryospheric dynamics of the MIZ...
in general, it is necessary to couple a sea ice model to an ocean model. Models of sea ice exist. An excellent review on sea ice growth, drift, and decay may be found in Hibler [1980a] and a review focusing on sea ice modeling in the Seasonal Sea Ice Zone (SSIIZ) may be found in Hibler [1980b]. As was pointed out by Wadhams [1980], the model equations and constitutive law for sea ice, used in the existing sea ice models, may have to be modified in order to reflect the special conditions and problems in the MIZ. Although they may be applicable, the simulations with existing sea ice models have a horizontal scale too large to resolve processes in the MIZ. The resolution of these models are a few hundred kilometers, whereas typical horizontal scales in the MIZ is only a few kilometers or less. Indeed, the total horizontal area with typical MIZ conditions are only about 200 km inward and outward from the ice boundary [Kozo and Tucker, 1974; Wadhams, 1980].

Another striking feature of the MIZ is the existence of a sharp and well defined ice edge. It has been common to attribute this fact to wind and waves. However, in a recent paper, Reed and O'Brien [1981] offered another explanation. They showed that there exists a geostrophic equilibrium for the sea ice medium (i.e., a balance between the Coriolis force and the internal stress force resulting from a viscous-plastic constitutive law of a sea ice). For typical ice parameters, the equilibrium shape of the sea ice concentration (i.e., the fraction of unit area covered by ice) was very sharp at the edge, changing from zero at the edge to 80% in less than a few kilometers. Their equilibrium solution also exhibited an ice edge jet in the ice to the left (northern hemisphere) when facing the ice edge.

In view of this, a coupled ice-ocean model is proposed in sections 2 and 3. As is common in sea ice models, the ice pack medium is rendered a continuum by treating the ice concentration as a continuous variable. Thus, the 'density' of the ice medium is the ice concentration multiplied by the actual density of sea ice. To render the continuum hypothesis valid, the unit area has to be large compared to the individual ice floes comprising the ice medium. The floe size in the MIZ varies from a few meters or less up to 1 km or more [Wadhams, 1981]. In the present model a grid scale of 1 km is used, which is large enough to render the continuum hypothesis valid, but small enough to resolve most of the processes in the MIZ. The suggested coupled model below is also, due to its inherent nonlinearity, capable of resolving features such as banding and streaming provided their spatial scales are large enough to be resolved by the model (e.g., the bandwidth has to be a few kilometers or more). The model is solved by numerical integration applying the method of characteristics with specified time and space intervals as well as ordinary finite difference approximations.

The results reveal that the previous mechanism for upwelling at ice edges is destroyed when the ice cover is allowed to move. With a wind conducive of upwelling according to previous models the present model produces a weak downwelling, whereas winds conducive of downwelling give upwelling. On the basis of geostrophic equilibrium solution of Reed and O'Brien [1981], a new mechanism of ice edge upwelling is suggested in which no atmospheric forcing is necessary.

The effect of the internal ice stress has been studied by neglecting the internal ice stress in the momentum equations for sea ice. It is shown that this stress, as parameterized in the proposed model, is important in order to form and maintain a sharp ice edge. However, the response of the ocean beneath is not appreciably affected. Thus, prediction of ice edge position as well as the ocean response may be successfully modeled without invoking the cumbersome internal ice stress.

In the present model the flow of ice and ocean is unbounded. However, artificial boundaries are imposed where the open boundary conditions described by Reed and Smedstad [1983] have been applied.

2. MODEL FORMULATION

Considered below is the response of a coupled ice-ocean system to wind forcing. The motion will be described relative to a Cartesian coordinate system (x, y, z) which rotates about the vertical z axis with a uniform angular velocity \( \omega \). Thus, \( \omega \) is the Coriolis parameter. The system is forced by a specified atmospheric pressure distribution, \( P_{atm}(x, y, t) \) which gives rise to a wind velocity at the surface given by

\[
W_x = akxVP_{atm}/\rho \omega
\]
Here, V = \( \mathbf{i} \partial \mathbf{v} + \mathbf{j} \partial \mathbf{v} \). The vectors \( \mathbf{i}, \mathbf{j}, \) and \( \mathbf{k} \) are unit vectors along the \( x, y, \) and \( z \) axes, respectively; \( \rho_i \) is the density of the air; and the ratio \( \alpha \) is positive and less than unity. This ratio depends on the surface boundary layer characteristic and may therefore in general depend on the fraction of unit area covered by ice or the ice concentration. A sketch of the model is provided by Figure 2.

2.1. The Sea Ice Model

The sea ice model of Reed and O'Brien [1981] is modified by adding the stresses at the interfaces ice/atmosphere and ice/ocean. The motion is assumed to be independent of \( y, \) the along-edge coordinate. This approximation seems to be supported by the large difference in across-edge and along-edge horizontal scales revealed by satellite images of the MIZ [e.g., Zwally and Gloersen, 1977] and also makes the model simple and tractable. Let \( u \) denote the horizontal ice velocity with components \( (u_x, u_y) \) corresponding to the coordinates \( (x, y) \) and \( A \) the ice compactness or ice concentration (i.e., the fraction of area covered by ice). Then the momentum equations may be written [see Hibler, 1979]

\[
\rho_i D A(u_x + u_{xx} - \mathbf{f}) = \sigma_x + A (\tau_{ux} + \tau_{uu})
\]

(2)

Here, \( \rho_i \) is the density of ice. \( D \) is the thickness of the ice defined as the mean thickness integrated over the fraction of unit area actually covered by ice and \( \sigma \) the internal ice stress. The vector \( \tau_{ux} \) with components \( (\tau_{ux}, \tau_{uy}) \) corresponding to the coordinates \( (x, y) \) represents the atmospheric drag exerted by the wind, \( \mathbf{W}_o \), on the ice and \( \tau_{uu} \), with components \( (\tau_{ux}, \tau_{uy}) \), a similar drag exerted by the ocean on the ice. In all the equations the subscripts \( x \) and \( t \) are to be interpreted as differentiation with respect to subscript.

There are two important differences between the equations above and those of Hibler [1980a]. First, the stress terms have been multiplied by the ice concentration. The reason is that the momentum transfer to the ice medium, as a continuum, is proportional to the fraction of area covered by ice. This is important since the ice concentration in the MIZ is allowed to approach zero at the ice edge. Second, the forcing term due to oceanic 'tilt' has been neglected. According to Hibler [1979], this term is important only on long time scales (i.e., a year or longer). Since the present model will focus on transient phenomena on a short time scale (i.e., a few days to a few weeks) the term is omitted.

The sea ice model is coupled to the ocean only by the interfacial drag \( \tau_{uu} \). It is important to realize that since the ice is floating there is no interaction in terms of hydrostatic pressure between the two media. Let \( U \) denote the oceanic upper layer velocity with components \( (U, V) \) along \( (x, y) \), respectively. Then the interfacial drag is parameterized by a linear drag according to the formula [McPhee, 1979; Hibler, 1980a]

\[
\tau_{uu} = \rho C_{uu} (U - u)
\]

(4)

Here \( \rho \) is the density of the sea water, \( C_{uu} \) a drag coefficient, and the turning angle is assumed to be zero. Thus, if \( |U| \) is larger than \( |u| \), it entails an acceleration of the ice due to ocean currents. The stresses exerted by the atmosphere are parameterized by

\[
\tau_{uu} = \rho C_{uu} |W_o| W_o
\]

(5)

where \( C_{uu} \) the drag coefficient over ice and water, respectively, in general depends on the ice concentration. The studies of atmospheric boundary layer characteristics in the Arctic indicates that the drag coefficient over pack ice is about twice the drag coefficient over open water [see, for instance, Feldman et

### Table 1 Standard Case Description

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H )</td>
<td>equilibrium depth of pycnocline</td>
<td>10^3</td>
<td>m</td>
</tr>
<tr>
<td>( \rho_{p,i} )</td>
<td>density of ice times thickness of ice</td>
<td>3 \times 10^3</td>
<td>kg/m^2</td>
</tr>
<tr>
<td>( \rho )</td>
<td>density of sea water</td>
<td>1 \times 10^3</td>
<td>kg/m^3</td>
</tr>
<tr>
<td>( k^* )</td>
<td>plastic strength</td>
<td>5 \times 10^3</td>
<td>N/m</td>
</tr>
<tr>
<td>( f )</td>
<td>Coriolis parameter</td>
<td>1.46 \times 10^{-4}</td>
<td>s^{-1}</td>
</tr>
<tr>
<td>( g^* )</td>
<td>reduced gravitational acceleration</td>
<td>0.02</td>
<td>m/s^2</td>
</tr>
<tr>
<td>( C_{d} )</td>
<td>drag coefficient at interface ice/water</td>
<td>8.6 \times 10^{-4}</td>
<td>m/s</td>
</tr>
<tr>
<td>( C_{d}/C_{ow} )</td>
<td>ratio of drag coefficient (air/ice)/(air/ocean)</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>( \kappa )</td>
<td>dispersivity coefficient of ice</td>
<td>20.0</td>
<td></td>
</tr>
<tr>
<td>( \alpha_{r,i} )</td>
<td>amplitude of wind stress air/ocean</td>
<td>-0.1</td>
<td>N/m^3</td>
</tr>
<tr>
<td>( \delta )</td>
<td>( u ) folding time of forcing</td>
<td>10^{-4}</td>
<td>s^{-1}</td>
</tr>
<tr>
<td>( \Delta x )</td>
<td>grid size</td>
<td>10^3</td>
<td>m</td>
</tr>
<tr>
<td>( \Delta t )</td>
<td>timestep</td>
<td>720</td>
<td>s</td>
</tr>
</tbody>
</table>

Forcing \( \tau_{uu} = \tau_{d} (1 - e^{-t / \tau_{d}}) \)
al., 1981]. In lack of more specific data, it is assumed in this study that $C_w$ is twice the value of the drag coefficient over open water (Table 1).

In the MIZ, most of the ice-ice interaction happens by bumping rather than grinding or scratching between individual floes. Therefore, a simplified version of the Hibler [1977, 1979] constitutive law, as described by Reed and O'Brien [1981], is used where the internal stress is given purely by an uncompensated pressure $\pi$. Moreover, it will be assumed that $\pi$ is a function of the areal mass $(m = \rho_i AD)$ only, such that a propagation speed $c$ of disturbances may be defined by

$$c^2(m) = dx/dm \quad (7)$$

Following Hibler [1979, 1980a] and Reed and O'Brien [1981], the following expression for the propagation speed will be used, viz.,

$$c^2 = c_0^2(\pi)^A \exp{[-\kappa(1 - A)]} \quad (8)$$

Here

$$c_0^2 = \kappa^2(\pi^* / \rho D_0)[\exp{(-\kappa)} + (\kappa - 1)] \quad (9)$$

is a typical propagation speed for 100\% concentration ($A = 1$), and $\pi^*$ is a typical value of the plastic strength (Table 1). The 'dispersivity' coefficient $\kappa$ is of the order of 15–20 for Arctic Sea Ice (Hibler, 1979). The relation (8) allows the plastic strength to be practically zero for low concentrations up to 80–85\%, but allows a rapid build up of strength as the concentration approaches 100\%; $D_0$ is a typical thickness for the inner ice pack ($\sim 3$ m).

With the definitions and assumptions above the momentum equations governing the motion of sea ice become

$$\frac{dx}{dt} + u_x + f_v + (c^2)A_x = \left(\frac{\tau_{sw} + \rho C_w(A - U - u)}{\rho D}\right) \quad (10)$$

and

$$\frac{dh}{dt} + u_x + f_v = \left(\frac{\tau_{sw} + \rho C_w(V - v)}{\rho D}\right) \quad (11)$$

The thermodynamics of sea ice will not be considered (i.e., melting and freezing of ice will be neglected). Thus, the continuity equation becomes

$$(AD)_x + (uAD)_h = 0 \quad (12)$$

The thickness is here assumed to vary on a scale large compared to the concentration and is therefore assumed to be constant. Then (12) reduces to

$$(AD)_x = 0 \quad (13)$$

The Ocean Model

The ocean model is a two-layer model, common in coastal upwelling problems [O'Brien and Hurlburt, 1972], in which the lower layer is assumed to be deep enough for the motion in the lower layer to be vanishing small. Such models are referred to as reduced gravity models. Let $h$ denote the thickness of the upper layer and $H$ the equilibrium thickness. Then the governing equations are the momentum equations

$$U_x - fV + gh_x = \left[\frac{[1 - A\tau_{sw} - A\tau_{sw}^*]}{\rho H}\right]$$

$$V_x + fU = \left[\frac{(1 - A)\tau_{sw} - A\tau_{sw}^*}{\rho H}\right] \quad (14)$$

and the continuity equation

$$h_x + HU_x = 0 \quad (15)$$

Here $g^2 = \Delta \rho / \rho$ is the gravitational acceleration, where $\Delta \rho$ is the density difference between the two layers. The stress, $\tau_{sw}$ with components $(\tau_{sw}, \tau_{sw}^*)$ is the atmospheric drag on the water. Note, that in the ice free regions $A = 0$ in which case the forcing is the stress $\tau_{sw}$ only. In ice-covered areas, the atmospheric forcing is assumed to depend on the fraction of open water (i.e., $(1 - A)$). The particular choice in (14) and (15) is chosen so that the transfer of atmospheric momentum to the ocean decreases with increasing compactness. The sign of the interfacial stress is such that if the speed of the ice is larger than that in the ocean, the ice will accelerate the ocean beneath it.

2.3. Boundary Conditions

The model is bounded by two open boundaries at $x = \pm L$ (see Figure 2). At these boundaries the open boundary conditions suggested by Reed and Smedstad [1983] are applied. These boundary conditions are based on those of Orlanski [1976] modified by Camer/engo and O'Brien [1980] but is extended to allow for forcing to be present at the open boundary.

A boundary of special interest is constituted by the ice edge (i.e., the position at which the ice concentration goes to zero). The actual position of the ice edge, $x = L(t)$, is not given a priori and must be solved as an integral part of the problem. The kinematic boundary condition at the ice edge provides an equation from which this position can be derived, viz.,

$$L = u \quad (x = L) \quad (17)$$

where $L$ denotes the derivative of $L$ with respect to time.

3. NUMERICAL PROCEDURE

The governing equations for the ocean as well as the ice will be solved by means of numerical integration, in which the governing equations (10–13) and (14–16) are approximated by finite differences. While integrating the oceanic part, the ice variables are kept fixed and vice versa.

3.1. Ocean

A nonstaggered grid is applied. The model grid size is 1 km. This grid size gives the model a resolution small enough to resolve the typical scales in the MIZ. In comparison, the internal radius of deformation is of the order of 10 km. A 12 min time step is incorporated by the model. The combination of grid size and time step is well within the constraints posed by the C.F.L. condition.

A forward differencing in time is used in (10), while (14) and (15) are backward differentiated except for the Coriolis term in (14). Sielecki [1968] is credited with developing this scheme. All single derivatives in space are replaced by standard second-order centered differences. The scheme is neutrally stable even when interfacial drag and lateral diffusion are neglected. Moreover, the scheme has no numerical dissipation when the C.F.L. condition is satisfied. Some numerical dispersion is present for the shortest waves. For a more detailed discussion of this type of scheme [see Martinson et al., 1979] who used the scheme for a similar system of equations in order to study storm surges along the western coast of Norway.

3.2. Sea Ice

The equations governing the response of the sea ice (i.e., (10), (11), and (13)) are nonlinear. They therefore call for another approach than above in order to be solved. The approach taken here is to solve them by means of the method of characteristics. Thus, the governing equations may be rewritten as outlined in Appendix A to give the compatibility equations, viz.,
The system (18H25) is solved utilizing fixed time and space intervals [Hartree, 1953; Uster, 1966]. The procedure closely follows that of O'Brien and Reid [1967]. The time step and grid intervals are the same as those used for the ocean.

The forcer is provided by an along edge jet in the ice medium into the paper, with maximum at the edge and decreasing toward the inner ice pack. Contour interval is 6 m.

The derivatives \( (D/dt)_x \) are to be evaluated along the characteristics defined by their slopes, viz.,

\[
C_+ \text{: } (Dx/dt) = u + c
\]

\[
C_- \text{: } (Dx/dt) = u - c
\]

and the derivative \( (D/dt)_x \) along the curves whose slope in the \((x, t)\) plane is

\[
C_+ \text{: } Dx/dt = u
\]

The system (18)-(25) is solved utilizing fixed time and space intervals [Hartree, 1953; Lister, 1966]. The procedure closely follows that of O'Brien and Reid [1967]. The time step and grid size are the same as those used for the ocean.

3.3. Boundary Conditions

At the artificially imposed boundaries, \( x = \pm l \) the open boundary conditions of Reed and Smedstad [1983] has been implemented.

The moving boundary constituted by the ice edge is computed from the finite difference representation of (17). At the ice edge \( A \) equals zero which entails that all the characteristics collapse into the one given by (25). The velocity components at the leading edge are found invoking (18) and (20) with \( A = 0 \), applying the procedure used for interior points.

3.4. Initialization

The integration area extends 250 km to each side of the initial position of the ice edge (origin) (see Figure 2). The response of the model from an equilibrium state is considered below. The ocean is spun up from rest, while the sea ice state is specified according to the geostrophic equilibrium solution of Reed and O'Brien [1981] (with \( \alpha = 0 \)). Table 1 lists other parameters used in the model experiments.

5. THE RESPONSE OF THE MODEL TO WIND FORCING

Next, consider the response of the model described above to a wind which by Gammelsroed et al. [1975] is conducive of upwelling. Accordingly, the model is forced by an atmospheric stress along the ice edge and to the right when facing the ice edge. In the previous analytical models of ice edge upwelling [Gammelsroed et al., 1975; Clarke, 1978] this forcing will generate an off-ice Ekman transport that is not met by a similar transport below the ice in the ice-covered region. This gives rise to a divergence in the oceanic transport across the ice edge, which in turn drives the upwelling.

The present model incorporates a moving ice cover. Note, that in the present model the atmospheric drag coefficient on ice, \( C_{aw} \), is chosen to be twice the drag coefficient on the ocean, \( C_{ow} \) (Table 1). Thus, the momentum from the atmosphere is more efficiently transferred to the ice than to the ocean. The response of the model is depicted by Figures 4 and 5 which are similar to Figure 3. Note, that positive anomalies entails a decrease in thickness of upper layer (i.e., upwelling).

When the wind is turned on, the ice starts to move in the wind direction (Figure 4), but as the ice speed increases, the
Coriolis force forces the ice to move to the right of the wind as well. Simultaneously, the ice concentration gradient close to the edge increases and forms an ice edge similar to the geostrophic equilibrium structure of Reed and O'Brien [1981]. The response of the ocean (Figure 5) shows no upwelling. Indeed, the expected upwelling is replaced by a weak downwelling. As will be discussed below, it is the created divergences and convergence in the across edge upper ocean transport which drives the pycnocline up or down and therefore determines the upwelling and downwelling.

6. THE EFFECT OF NEGLECTING INTERNAL ICE STRESS

As pointed out by Hibler [1979] the internal ice stress is particularly important in areas where there exist barriers in the form of a coast, etc. In the open ocean, and in the MIZ in particular, this term may not be all that important. As was pointed out by Thorndike and Colony [1982], most of the ice motion in response to geostrophic winds can be explained by a simple balance between the stresses exerted by the atmosphere and ocean and the Coriolis force. It is worthwhile to note that according to Hibler [1981] analysis of idealized plastic ice systems show that the findings of Thorndike and Colony [1982] also can be explained by increases plastic strength in the ice-ice interaction term. It is, therefore, of some interest to perform an experiment in which the internal ice stress in (10) and (11) are neglected. Since the internal stress is responsible for the nonlinear character of the sea ice model also the advection terms may be omitted in this case. In these circumstances the laborious method of characteristics may be replaced by a scheme similar to the scheme used for the oceanic part. Note that in this case the ice as well as the ocean is spun up from rest.

By applying a wind stress equal to that of the previous section the effect of neglecting the internal ice stress may be investigated. The evolution of ice concentration and pycnocline deviation is shown in Figures 6 and 7. The results may be compared with those of the previous section as depicted by Figures 4 and 5. The displacement of the leading ice edge (Figures 6) as well as the structure of the oceanic downwelling (Figure 7) is very similar in the two cases. The detailed structure of the time and space variation of the ice concentration is different. The strong increase in the gradients in ice concentration close to the edge is missing. The sharp ice edge as displayed by Figure 4 is therefore caused and maintained by the internal ice stress. The gradual increase in the ice concentration gradients in Figure 6 is caused by the interfacial stress alone. This is, however, a slow and weak process.

The detailed structure of the downwelling as depicted by Figures 5 and 7 in the two cases, respectively, is only slightly different. The small difference is due to the different ice concentration distribution. The dynamics of the downwelling is the same and may be understood by analyzing the upper ocean transports. The upper layer across edge velocity component in the ocean is shown in Figure 8. It is everywhere negative, which means that water in the upper layer is transported from the ice-covered region across the ice edge towards open water. The amplitude reaches its maximum toward the inner ice pack and decreases steadily toward the ice edge and the open water. Thus, it gives rise to a convergence of the upper layer of the ocean with a broad maximum at the edge and decreasing to either side. It is this convergence which in turn drives the downwelling depicted by Figures 5 and 7. As pointed out above, the momentum from the atmosphere is allowed to be transmitted more effectively to the ice than to the ocean. In this case then, in the ice covered region, the momentum from the ice to the ocean, through the interfacial stress term, gives rise to an upper layer transport in the ocean beneath that is larger than the off-ice transport created by the wind over open water. It seems, therefore, clear that the dynamics of the upwelling and downwelling at ice edges depends crucially upon the motion of the ice rather than on the actual distribution of ice in terms of ice concentration.

In this simulation and the two previous ones, no noise from the open boundaries is apparent. A more detailed study of the pycnocline deviation (not reported here) did not reveal any appreciable noise even close to the open boundaries. It is important to realize that the forcing was uniform in space throughout the experiments. Thus, the applied open boundary condition suggested by Reed and Smedstad [1983] seems to work well in these circumstances.

7. AN ANALYTICAL INTERPRETATION

The results of the previous sections suggests that the upwelling and downwelling dynamics in the present model may be understood by analysis of a fairly simple analytical model. As shown above, the internal ice stress and hence the nonlinear
advection terms are not crucial for the upwelling dynamics. This justifies a simple analytical ice model governed by the momentum equations [Nansen, 1902]

\[ u_i = f_0 - \rho C_{uw} u / \rho_1 D \]  
\[ v_i = -fu - \rho C_{uw} v / \rho_1 D + \tau / \rho_1 D \]  

Here, also, the oceanic influence is neglected, since, based on the previous results, the velocities differ by an order of magnitude. Assuming the atmospheric stress, \( \tau \) to be a function of time only, no space dependence will be generated within the ice pack. Thus, the continuity equation (13) states that \( A \) is conserved (i.e., if \( A \) = constant, initially it will stay constant except right at the leading edge where it abruptly drops to zero). The leading edge, however, will move in accordance with (26) and (27). The solution to (26) and (27) are the well-known damped inertial oscillations. Thus for large times

\[ u \sim u_0 \]
\[ v \sim \left( \frac{\alpha}{f} \right) u_0 \]
\[ L \sim u_0 t \quad x > L \]

where

\[ u_0 = \left[ \frac{\tau}{\rho_2 \rho D} \right] \left[ 1 + \left( \frac{\alpha}{f} \right)^2 \right]^{-1} \]
\[ \alpha = \rho C_{uw} / \rho_1 D \]  

Neglecting inertial oscillation and again assuming the ocean velocities to be small, the momentum equations for the ocean may be written

\[ 0 = fV - g^2 h_x + \gamma uAH(x - L) \]  
\[ V_x = -fu + \gamma vAH(x - L) \]  

whereas the continuity equation (16) is unchanged. Here, \( H(x - L) \) is the heavyside function defined by

\[ H(x-L) = \begin{cases} 1 & x > L \\ 0 & x < L \end{cases} \]  

For convenience, the constant

\[ \gamma = C_{uw} / H \]  

is introduced. No atmospheric forcing is applied to the open ocean. This is based on the previous assumption that the momentum transfer to the ice is much more efficient than the momentum transfer to the ocean. This will serve to enhance the response of the ocean. By a simple transformation to a new coordinate system moving with the ice edge (30)-(32) may be solved, assuming that the dependent ice variables have attained their values for large time as given by (28). Details of the method is given in Appendix B. The solution in terms of the thickness of the upper layer is

\[ h(x, t) = H - (g \gamma u_0 / 2 f^2 \lambda t) \exp \left[ -|x - u_0 t| / \lambda \right] \]  

where \( \lambda \) is the deformation radius, viz.,

\[ \lambda = (g^2 H)^{1/2} / f \]  

To compare the structure of this solution to the previous numerical solution, Figure 9 is constructed from (34). Values given to the parameters are the same as in the previous solutions. Although the detailed structure is different, it is obvious that the main dynamics are retained in this analytical model. The similarity between Figure 9 and Figures 5 and 7 is striking. Owing to the omission of the atmospheric stress in the open ocean for the analytical model, the response as depicted by Figure 9 is enhanced. The omitted stress will serve to decrease its amplitude, while inclusion of a variable ice concentration will serve to broaden the downwelled region. This solution

\[ \frac{\partial h}{\partial t} + \frac{\partial (hV)}{\partial x} = -g \frac{\partial h}{\partial x} \]

\[ \frac{V_x}{V} = -fu + \gamma vAH(x - L) \]  

This solution is obtained by neglecting inertial oscillations.
Numerical models may be understood in terms of the divergence and convergence of the across ice edge oceanic transports in the welling dynamics as revealed by the much more complex analytical model in a fashion similar to Figure 7. This figure may be enhanced in the analytical case. The general response of the ocean in the different cases are strikingly similar, although the response is enhanced in the analytical case.

Therefore supports the conclusion that the upwelling and downwelling dynamics as revealed by the much more complex numerical models may be understood in terms of the divergence and convergence of the across ice edge oceanic transports in the upper layer.

8. DISCUSSION

A dynamical coupled ice-ocean numerical model has been developed in order to study upwelling dynamics in the marginal ice zone of the world oceans. The sea ice is rendered a continuum by treating the ice compactness or fraction of a unit area covered by ice as a continuous variable. The ice compactness is allowed to approach zero, the sea ice model is based on the ice strength considerations by Hibler [1979]. Because of the internal ice stress term and the fact that the ice concentration is allowed to approach zero, the sea ice model is nonlinear. In the one-dimensional case treated in the present paper, the governing equations of the sea ice are solved by the method of characteristics with specified time and space intervals [O'Brien and Reid, 1967]. The sea ice model is coupled to the ocean below by an interfacial stress on the interface ice-ocean. This stress is proportional to the velocity difference between the ice and the ocean [McPhee, 1979]. Because of the possibility of the sea ice to be in geostrophic equilibrium [Reed and O'Brien, 1981] the sea ice may provide a forcing mechanism for ice edge upwelling without incorporating any external forces (section 4). The models discussed in sections 5 and 6 are forced by an atmospheric drag on both ice and ocean. Since the ice only occupies a fraction of a unit area, the momentum transfer to the ice is proportional to the ice concentration, whereas the momentum transfer to the ocean is proportional to the fraction of open water. Also, the transfer of momentum to the ice is more efficient than that to the ocean, which is reflected by a larger drag coefficient atmosphere-ice than atmosphere-ocean.

The motion of the ice plays a crucial role. The response of the ocean in cases where one, based on previous works on ice edge upwelling, would expect upwelling to take place, is replaced by a weak downwelling. Thus, contrary to expectations, it is winds which blow along the ice edge and to the left when facing the ice edge which favors upwelling. This is perhaps most clearly shown by the simple analytical model suggested in section 7 (34) which enhances the motion of the main pycnocline. Note that this solution is symmetric when the wind is reversed.

It is also interesting to note that some of the theoretical findings of this paper are supported by observations from the Greenland Sea MIZ as reported by the NORSEX group [Johannessen et al., 1983].

APPENDIX A: DERIVATION OF COMPARABILITY AND CHARACTERISTIC EQUATIONS

Multiplying (13) by a constant \( \mu \) and adding the result to (10) gives

\[
\begin{align*}
  u_x + (u + \mu A) u_x + \mu [A_x + (u + c^2/\mu A) A_x] = f_0 + [\tau_a^* + \rho C_w(U - u)]/\rho D
\end{align*}
\]

(Eq. A1)

Equating the coefficients in front of \( u_x \) and \( A_x \) follows

\[
\frac{\mu}{A} = \pm \frac{c}{A}
\]

(Eq. A2)

Thus, (A1) may be written

\[
\begin{align*}
  \left( \frac{Du}{dt} \right)_z \pm \frac{c}{A} \left( \frac{Da}{dt} \right)_z = f_0 + [\tau_a^* + \rho C_w(U - u)]/\rho D
\end{align*}
\]

(Eq. A3)

and

\[
\begin{align*}
  \left( \frac{D\xi}{dt} \right)_z = u \pm c
\end{align*}
\]

(Eq. A5)

Equation (11) may be rewritten in a similar fashion, viz.,

\[
\begin{align*}
  \left( \frac{Du}{dt} \right)_z = -fu + [\tau_a^* + \rho C_w(V - v)]/\rho D
\end{align*}
\]

(Eq. A6)

\[
\begin{align*}
  \left( \frac{D\xi}{dt} \right)_z = u
\end{align*}
\]

(Eq. A7)

APPENDIX B: DETAILS OF THE ANALYTICAL SOLUTION

Defining the transformations

\[
\begin{align*}
  \xi = (x - u_0 t)/\lambda
\end{align*}
\]

(Eq. B1)

and

\[
\begin{align*}
  \eta = u_0 t/\lambda
\end{align*}
\]

(Eq. B2)

where

\[
\begin{align*}
  \lambda = (g*H)^{1/2}f
\end{align*}
\]

(Eq. B3)

(30) and (31) become

\[
\begin{align*}
  0 = f V - (g^*\lambda) h_\zeta + \gamma u_0 \tilde{A}(\xi)
\end{align*}
\]

(Eq. B4)
\[ V_x \quad V_z = - \frac{f}{u_0} U + \frac{\nu}{f} \lambda \frac{\partial \Psi}{\partial \xi} \quad (B1) \]

\[ h_t - h_x = - \frac{H}{u_0} U_t \quad (B6) \]

These equations may be solved in two regions: region I, \( \xi < 0 \), and region II, \( \xi > 0 \), and matched at \( \xi = 0 \) by requiring the upper layer, \( h \), to be continuous at that position. By eliminating \( h \) and \( V \) from \((B4) - (B6)) \) follows

\[ U_{ac} - U_t = 0 \quad \xi < 0 \quad (B7) \]

\[ U_{ac} - U_u = - \frac{\nu}{f^2} u_0 \quad \xi > 0 \quad (B8) \]

In order for \( U_t \) and \( U_u \) to stay bounded everywhere, the solutions to these equations may be written

\[ U_t = C_1(\eta) e^\xi \quad (B9) \]

and

\[ U_u = C_2(\eta) e^{-\xi} + \frac{\nu}{f^2} u_0 \quad (B10) \]

From the requirement that \( U \) be continuous at \( \xi = 0 \) follows,

\[ C_1 = C_2 + \frac{\nu}{f^2} u_0 \quad (B11) \]

Thus, \((B6)) gives

\[ h_t - h_x = - \frac{H}{u_0} C_1(\eta) e^\xi \quad (B12) \]

and

\[ h_{ac} - h_{ac} = + \frac{H}{u_0} C_2(\eta) e^{-\xi} \quad (B13) \]

The solution to \((B12)\) and \((B13)\) may be written

\[ h_t = H + F_1(\eta) e^\xi \quad (B14) \]

and

\[ h_u = H + F_2(\eta) e^{-\xi} \quad (B15) \]

By applying the condition \( h_t = h_u \) at \( \xi = 0 \) follows

\[ F_1(\eta) = F_2(\eta) = F(\eta) \quad (B16) \]

and since \( h_t = h_u = H \) initially also

\[ F(0) = 0 \quad (B17) \]

Substitution of \((B14)\) and \((B15)\) into \((B12)\) and \((B13)\) utilizing \((B16)\) gives

\[ F_x - F = - \frac{H}{u_0} C_1 \quad (B18) \]

and

\[ F_y + F = - \frac{H}{u_0} C_2 \quad (B19) \]

By adding these two equations and invoking \((B17)\) follows

\[ F_x = \frac{H}{u_0} (C_2 - C_1) \quad (B20) \]

or invoking \((B17)\) that

\[ F = - \frac{1}{2} \frac{\nu}{f^2} H \quad (B21) \]

Equation \((34)\) follows by substituting \((B21)\) into the expressions \((B14)\) and \((B15)\) and transform back to the original system \((x, t)\).

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