

## Hybrid Vertical Coordinate Grid Generator

Bleck (2002) describes the hybrid vertical coordinate grid generator along with its initial implementation in a global version of HYCOM. The following description also describes the substantial modification to this algorithm that has occurred since then.

### 1. Vertical Coordinate Adjustment in the Open Ocean

The algorithm controlling the transition between isopycnic and level ( $p$ ) coordinates in the open ocean works as follows: Consider three consecutive isopycnic layers labeled 0, 1, and 2 in a stratified water column where the user has set the minimum thicknesses to  $p_0$ ,  $p_1$ , and  $p_2$  respectively. Suppose that specific volume  $\mathbf{a}_1$  differs from its isopycnic reference value  $\hat{\mathbf{a}}_1$ . To restore isopycnic conditions, it is necessary to re-discretize the water column in a manner that preserves the overall height of the column, represented by the integral  $\int \mathbf{a} dp$ , while changing  $\mathbf{a}_1$  to  $\hat{\mathbf{a}}_1$ . Conservation of the integral requires that one or more layer interfaces must be relocated to different pressure levels. If layer 1 is too dense ( $\mathbf{a}_1 < \hat{\mathbf{a}}_1$ ), the upper interface is moved upward to transfer less-dense water from layer 0 to layer 1. If layer 1 is too light ( $\mathbf{a}_1 > \hat{\mathbf{a}}_1$ ), the lower interface is moved downward to transfer denser water from layer 2 to layer 1. This method does not work for the model layer in contact with the bottom if it is too light. A special algorithm described below is included to handle this case by extruding water into the layer above.

The two cases  $\mathbf{a}_1 < \hat{\mathbf{a}}_1$  and  $\mathbf{a}_1 > \hat{\mathbf{a}}_1$  are discussed separately. For the first case where the layer is too dense ( $\mathbf{a}_1 < \hat{\mathbf{a}}_1$ ), the upper interface is moved upward and mass is exchanged between layers 0 and 1. Conservation of  $\int \mathbf{a} dp$  requires that the upper interface be moved to the new pressure interface

$$\hat{p}_1 = \frac{p_1(\mathbf{a}_0 - \mathbf{a}_1) + p_2(\mathbf{a}_1 - \hat{\mathbf{a}}_1)}{\mathbf{a}_0 - \hat{\mathbf{a}}_1}. \quad (1)$$

Since the weight assigned to  $p_2$  is negative, (1) will not necessarily yield a solution  $\hat{p}_1 > p_0$  for large differences between  $\mathbf{a}_1$  and  $\hat{\mathbf{a}}_1$ . This is no problem because the minimum thickness constraint is applied to layer 0 by replacing  $\hat{p}_1$  by

$$\tilde{p}_1 = \min \left[ p_2, \max(\hat{p}_1, p_0 + \Delta_0) \right], \quad (2)$$

where  $\Delta_0$  is a specified minimum layer thickness. Of course, moving the interface to  $\tilde{p}$  instead of  $\hat{p}$  no longer permits isopycnic conditions to be restored.

Following Bleck and Benjamin (1993), the user chooses the absolute minimum thickness  $\mathbf{d}_0$ . The actual minimum thickness  $\Delta_0$  is then calculated in a manner that insures a smooth transition between the isopycnic and non-isopycnic domains. The function  $\Delta_0$  is determined by a continuously differentiable ‘‘cushion’’ function, which for large positive arguments  $\Delta p \equiv \hat{p}_1 - p_0$  returns the argument  $\Delta p$  (meaning that  $\tilde{p}_1 = \hat{p}_1$ ) and for large negative arguments returns  $\mathbf{d}_0$ :

$$\Delta_0 = \left\{ \begin{array}{ll} \mathbf{d}_0 & \text{if } \frac{\Delta p}{\mathbf{d}_0} \leq q_{\min} \\ \left( \frac{\mathbf{d}_0}{q_{\max}} \right) \left( \frac{\Delta p}{\mathbf{d}_0} \right) \left\{ 1 + C_a \left[ 1 - C_b \left( \frac{\Delta p}{\mathbf{d}_0} \right) \right]^2 \right\} & \text{if } q_{\min} < \frac{\Delta p}{\mathbf{d}_0} < q_{\max} \\ \Delta p & \text{if } \frac{\Delta p}{\mathbf{d}_0} \geq q_{\max} \end{array} \right\}, \quad (3)$$

where the constants are given by

$$C_a = \frac{q_{\min}^2 (q_{\max} - 1)}{(q_{\max} - q_{\min})^2}, \quad C_b = q_{\min}^{-1}. \quad (4)$$

The limits  $q_{\min}$  and  $q_{\max}$  control the width of the transition zone between isopycnic and  $z$  coordinates, and are set to default values of  $q_{\min} = -2$  and  $q_{\max} = 4$ . When the upper interface must be raised, lighter water is entrained to increase  $\mathbf{a}_1$  to a value  $\tilde{\mathbf{a}}_1$ :

$$\tilde{\mathbf{a}}_1 = \frac{\mathbf{a}_1(p_2 - p_1) + \mathbf{a}_0(p_1 - \tilde{p}_1)}{p_2 - \tilde{p}_1}. \quad (5)$$

However, application of the minimum thickness requirement can make it necessary for the upper interface to be lowered ( $\tilde{p}_1 > p_1$ ) and thus modify layer 0. Conservation of  $\int \mathbf{a} dp$  then leads to

$$\tilde{\mathbf{a}}_0 = \frac{\mathbf{a}_1(\tilde{p}_1 - p_0) + \mathbf{a}_0(p_1 - p_0)}{\tilde{p}_1 - p_0}. \quad (6)$$

The preservation of minimum layer thickness by increasing the thickness of layer 0 always overrides attempts to restore isopycnic conditions.

For the second case where layer 1 is too light ( $\mathbf{a}_1 > \hat{\mathbf{a}}_1$ ), the lower interface is moved downward and mass from layer 2 is entrained into layer 1. Conservation of  $\int \mathbf{a} dp$  yields the new pressure level to which the interface must be relocated:

$$\hat{p}_2 = \frac{p_1(\hat{\mathbf{a}}_1 - \mathbf{a}_1) + p_2(\mathbf{a}_1 - \mathbf{a}_2)}{\hat{\mathbf{a}}_1 - \mathbf{a}_2}. \quad (7)$$

To maintain the minimum thickness of layer 2  $\hat{p}_2$  is replaced by

$$\tilde{p}_2 = \min(\hat{p}_2, p_3 - \Delta_2), \quad (8)$$

where  $\Delta_2$  is determined from the cushion function

$$\Delta_2 = \left\{ \begin{array}{ll} \mathbf{d}_2 & \text{if } \frac{\Delta p}{\mathbf{d}_2} \leq q_{\min} \\ \left( \frac{\mathbf{d}_2}{q_{\max}} \right) \left( \frac{\Delta p}{\mathbf{d}_2} \right) \left\{ 1 + C_a \left[ 1 - C_b \left( \frac{\Delta p}{\mathbf{d}_2} \right) \right]^2 \right\} & \text{if } q_{\min} < \frac{\Delta p}{\mathbf{d}_2} < q_{\max} \\ \Delta p & \text{if } \frac{\Delta p}{\mathbf{d}_2} \geq q_{\max} \end{array} \right\}. \quad (9)$$

with  $\Delta p \equiv p_3 - \hat{p}_2$ . The following additional limitations are imposed on  $\tilde{p}_2$ :

$$\tilde{p}_2 = \min \{ p_1 + \max(\tilde{p}_2 - p_1, 1), \max[(p_1 + p_2)/2, p_3 - 2] \}. \quad (10)$$

If layer 1 is the deepest layer with nonzero thickness and it is too light, interface 2 cannot move downward to restore isopycnic conditions and a special algorithm must be executed: Since only interface 1 can be moved, this interface is moved downward to restore isopycnic conditions in layer 1. An “unmixing” algorithm is employed to achieve this goal. The water in layer 1 must be restratified into two sublayers such that the density of the upper sublayer exactly equals the density of layer 0, the density of the lower sublayer is close to the desired reference density, and the density averaged over the two sublayers equals the original layer density. Given

$$q = \frac{\mathbf{a}_1 - \hat{\mathbf{a}}_1}{\mathbf{a}_0 - \hat{\mathbf{a}}_1},$$

interface 1 is relocated using

$$\hat{p}_1 = p_1 + q(p_2 - p_1). \quad (11)$$

Thermodynamical variables in the lower sublayer are then calculated using

$$\hat{T}_1 = T_1 - \left( \frac{q}{1-q} \right) (T_1 - T_0) \quad (12)$$

for  $T$ , and the same equation for  $S$  and  $\sigma$ . (Actually, two of the thermodynamical variables are diagnosed using (12) with the third estimated from the equation of state). The closeness of lower sublayer density to the reference density is sacrificed if necessary to achieve two goals: (1) to prevent the thickness of layer 1 from decreasing by more than 50% using

$$\tilde{p}_1 = \min(\hat{p}_1, p_2 - \Delta_2), \quad (13)$$

where  $\Delta_2 = (p_2 - p_1)/2$ , and (2) to prevent runaway changes in  $T$  and  $S$  using

$$|T_1 - T_0| \leq |T_0 - T_{-1}| \quad (14)$$

for  $T$  and the analogous equation for  $S$ .

## 2. Effects of Cabbeling and the Nonlinear Equation of State

In practice, if  $T$  and  $S$  are fluxed across the relocated interfaces, then perfect restoration of isopycnic conditions is not possible ( $\tilde{\sigma}_1 \neq \hat{\sigma}_1$ ) due to the nonlinear equation of state. For this reason, HYCOM allows the user to choose which two of the three thermodynamical variables  $T$ ,  $S$ , and  $\sigma$  are fluxed across the moving interface. In each case, the third variable is calculated using the equation of state. When  $\sigma$  is one of the fluxed variables, then cabbeling is not a problem. However, since either  $T$  or  $S$  are no longer conserved in this case, it is desirable to minimize the negative influence of cabbeling to allow  $T$  and  $S$  to be fluxed across relocated interfaces. To achieve this, the following iterative procedure is included to insure in most cases that  $|\tilde{\mathbf{a}}_1 - \hat{\mathbf{a}}_1|$  is smaller than a prescribed tolerance after applying the grid generator. The new  $T$  value that results from raising the interface is

$$\tilde{T}_1 = \frac{p_1 - \hat{p}_1}{p_2 - \hat{p}_1} (T_0 - T_1). \quad (15)$$

With  $\tilde{S}_1$  estimated in the same manner, a new estimate of  $\tilde{\mathbf{a}}_1$  is made using the model equation of state. If  $|\tilde{\mathbf{a}}_1 - \hat{\mathbf{a}}_1|$  exceeds the required tolerance, then  $\hat{p}_1$  is recalculated using

$$\hat{p}_1 = \hat{p}_1 + \frac{\tilde{\mathbf{a}}_1 - \hat{\mathbf{a}}_1}{\mathbf{a}_0 - \hat{\mathbf{a}}_1} (p_2 - \hat{p}_1) \quad (16)$$

In this step,  $\hat{p}_1$  is not permitted to exceed  $p_1$ . The resulting specific volume  $\tilde{\mathbf{a}}_1$  is then re-calculated, and if the prescribed tolerance is not achieved, then  $\hat{p}_1$  is re-calculated by the same procedure. The procedure is repeated up to five times if necessary.

Although uncommon, there is a more significant problem that can arise from the nonlinear equation of state when  $T$  and  $S$  are fluxed across relocated interfaces. During HYCOM development, cabbeling resulting from vertical coordinate adjustment (in conjunction with  $T$  and  $S$  horizontal advection and diffusion) caused excessive interface relocation in limited regions such as beneath the Mediterranean salt tongue. The nonlinear equation of state can, for certain  $T, S$  profiles, produce either insignificant density changes or density changes of the wrong sign within a model layer when the grid generator relocates either the upper or lower interface. Repeated application of the grid generator then produces unacceptably large vertical coordinate migration. Code is therefore included in HYCOM to suppress the vertical coordinate adjustment when the change in  $\mathbf{a}_1$  does not have the expected sign or when excessive coordinate migration would be necessary to restore isopycnic conditions.

### 3. The Complete Adjustment Algorithm Governing the $p$ -Isopycnic-Sigma Coordinate Transitions

The complete hybrid coordinate adjustment algorithm in HYCOM also includes the transition between the open-ocean  $p$  and isopycnic coordinates to sigma coordinates in shallow water and back to  $p$  coordinates in very shallow water. This is implemented in a very simple manner. First, the user specifies the number of model layers  $N_s$  that are to become sigma coordinates along with the absolute minimum thickness  $\mathbf{d}_{\min}$  that is permitted for the sigma coordinates. These are specified in addition to the user-specified open-ocean minimum thicknesses  $\mathbf{d}_k$  for layer  $k$ . The minimum thickness of the sigma coordinates is constant for all model layers and is given by

$$\mathbf{d}_s = \frac{D}{N_s}, \quad (17)$$

where  $D$  is the total water depth.

The full grid generator is then implemented by calculating the following minimum thickness value for each model layer:

$$\tilde{\mathbf{d}}_k = \max[\mathbf{d}_{\min}, \min(\mathbf{d}_k, \mathbf{d}_s)]. \quad (18)$$

In a given model layer, the transition to sigma coordinates occurs where the water depth becomes sufficiently shallow to make  $\mathbf{d}_s < \mathbf{d}_k$ . The transition back to level coordinates occurs where the

water depth becomes sufficiently shallow to make  $\mathbf{d}_s < \mathbf{d}_{\min}$ . Thus, the proper coordinate transformation is assured if  $\mathbf{d}_k$  is replaced by  $\tilde{\mathbf{d}}_k$  before executing the vertical coordinate adjustment. Only the upper  $N_s$  layers will transition to sigma coordinates. Deeper layers will collapse to zero thickness at the bottom.

#### 4. Implementation Issues

When the hybrid grid generator is called, it is executed separately at each grid point. Thermodynamical variables are adjusted first at the pressure grid points. Before adjusting the vertical coordinates, the initial one-dimensional profiles of temperature, salinity, and density, plus the one-dimensional array of interface pressures, are saved. If the primary Kraus-Turner mixed layer model is selected, an unmixing algorithm must be performed so that the thermodynamical adjustments are consistent with this model. The model layer containing the mixed layer base is divided into two sublayers, and the mixed layer base is temporarily considered to be an additional vertical coordinate. Thermodynamical variables in the two sublayers are then estimated using the same “unmixing” algorithm that is used in the KTA model.

Once the profiles are saved, the vertical adjustment of vertical coordinates is performed at the pressure grid points using the previously outlined procedures. The subsequent adjustments of model thermodynamical variables and momentum must conserve their vertically averaged values and, for the thermodynamical variables, restore density as close to the isopycnic reference value as possible. The classic donor cell scheme satisfies these criteria, but has the undesirable property of being directionally dependent. For example, consider a sequence of model interfaces that have been moved downward to increase the density of intervening layers. If variables are mixed across the relocated interfaces from the top down, the mixing that previously occurred in layer  $k - 1$  does not influence the mixing in layer  $k$ . If variables are mixed from the bottom up, however, the mixing that previously occurred in layer  $k + 1$  does influence the mixing in layer  $k$ .

The algorithm included in HYCOM remaps each variable from the old to the new vertical grid in a manner that satisfies the two conditions mentioned above, but that is not directionally dependent. Given interface pressures  $p_k$  on the old grid and  $\tilde{p}_k$  on the adjusted grid, where  $K$  is the number of model layers, the old temperature profile is mapped onto the new adjusted vertical coordinates using

$$\tilde{T}_k = \frac{1}{\tilde{p}_{k+1} - \tilde{p}_k} \sum_{k=1}^K T_k \left\{ \max \left[ \tilde{p}_k, \min \left( p_{k+1}, \tilde{p}_{k+1} \right) \right] - \min \left[ \tilde{p}_{k+1}, \max \left( p_k, \tilde{p}_k \right) \right] \right\}. \quad (19)$$

Note that  $K$  must be increased by one when the KTA mixed layer model is used to account for the two sublayers within the layer containing the mixed layer base. In practice, the summation is performed between  $k = k_1$  to  $k = k_2$  in order to eliminate layers on the old vertical grid that do not overlap layers on the new grid. The downside to this procedure occurs in a model layer  $k$  if the interface  $k$  above is relocated upward and the interface  $k + 1$  below is relocated downward. In this case, the simultaneous mixing across both interfaces will prevent perfect restoration of the isopycnic reference density in layer  $k$ . However, repeated application of this procedure will tend

to drive the density of layer  $k$  toward its reference density. This fact coupled with the absence of directional dependence support our choice of (19) to remap variables on the adjusted grid in lieu of a one-way donor cell scheme.

Donor-cell vertical advection schemes are known to produce numerically induced diffusion. For this reason, the re-mapping scheme has been further modified at the Naval Research Laboratory by A. J. Wallcraft to use the higher-order PLM algorithm within the nearsurface non-isopycnic domain where the distances that interfaces are relocated are relatively large, and numerical diffusion is thus expected to be large. In the isopycnic interior where mixing is very weak, the original remapping algorithm is retained because the PLM algorithm has the undesirable effect of changing values of layer variables within a layer when fluid is only detrained from that layer. Tests have validated that this undesirable effect is outweighed by the positive effect of numerical diffusion reduction.

After adjusting the thermodynamical variables at pressure grid points, the momentum components are then adjusted on the  $u, v$  grid points in a manner that conserves vertically averaged momentum. The old and new vertical coordinates obtained at pressure grid points are first interpolated to the  $u$  grid points. The adjustment of  $u$  is then performed using an equation of the form (19). The same procedure is used to update  $v$  at the  $v$  grid points.

## REFERENCES

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- Bleck, R. and S. Benjamin, 1993: Regional weather prediction with a model combining terrain-following and isentropic coordinates, Part 1: Model description. *Mon. Wea. Rev.*, **121**, 1770-1785.